MAKING SENSE OF MATHEMATICS THROUGH NUMBER TALKS: A CASE
STUDY OF THREE TEACHERS IN THE ELEMENTARY CLASSROOM

by

MIRANDA GAYLE WESTBROOK

A Dissertation Submitted to the Faculty
in the Curriculum and Instruction Program
of Tift College of Education
at Mercer University
in Partial Fulfillment of the
Requirements for the Degree

DOCTOR OF PHILOSOPHY

Atlanta, GA
2019
DEDICATION

To my loving husband, Chad, I am forever indebted to the unconditional love, support, and patience you bestowed as I engaged in this endeavor. You always encouraged me to persevere, and together, we transformed this dream into a reality. I am truly blessed to have you in my life.

To my mom and dad, you inspired me to follow my dreams and taught me the value of hard work. The morning phone calls were always uplifting, and I love you both with all my heart.

To Brenda, thank you for understanding and supporting me on this journey.

To Linda, you were always confident that I would achieve this goal. Your enthusiasm and interest in my work gave me the strength I needed to keep writing.

To Lucas and Miranda, thanks for checking on my progress each day. Although it seemed impossible at times, you never doubted that I would succeed.

To Brian, thanks for listening and answering my questions along the way.

To Michelle and Ashley, thanks for all the laughs. Every day is truly an adventure.

To all my family, friends, and colleagues, thank you for always being there and offering your support.
ACKNOWLEDGMENTS

To my committee, Dr. Lacefield, Dr. Randolph, and Dr. Hall, thank you for your patience, kindness, and encouragement throughout this entire process. Your feedback and support were invaluable, and I couldn’t have accomplished this task without you. Thank you for believing in me.

To my editor, Elizabeth, I cannot thank you enough for all of your help along the way. Your prompt responses allowed me to move forward with my work without hesitation. I couldn’t have done it without you!

To my research participants, thank you so much for inviting me into your classrooms. Without you, this study would not have been possible. Your love for teaching inspired me, and I enjoyed learning from each of you.
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ABSTRACT

MIRANDA GAYLE WESTBROOK
MAKING SENSE OF MATHEMATICS THROUGH NUMBER TALKS: A CASE STUDY OF THREE TEACHERS IN THE ELEMENTARY CLASSROOM
Under the direction of William O. Lacefield, III, Ed.D.

Mathematics instruction in the United States has historically consisted of procedures and rote learning practices. Reform efforts in mathematics education support a conceptual approach that integrates reasoning and understanding of problems. Classroom number talks nurture the essence of learning in mathematics. The purpose of this study was to investigate the role of number talks in developing number sense and number relationships in third-grade students pursuant to teacher perception. This study also explored the instructional practices of teachers during number talks, using Parrish’s (2011, 2014) five fundamental tenets. An analysis of the types of questions that teachers pose during classroom number talks to elicit student responses was also conducted. An embedded, multiple-case study design was used to examine participant perceptions through a within-case analysis and a cross-case analysis. Three teacher participants in a large suburban school district in the southeastern United States served as the primary units of analysis, while the students in their classrooms served as the subunits of analysis. Over the course of a six-week period, each participant completed four, audio-recorded interviews and three video-recorded observations. The findings suggested that classroom number talks influence students’ number sense understanding by encouraging them to
verbally reason about their thinking. Conclusions were (a) number talks promote accurate and precise communication about mathematics; (b) the learning environment is critical to the success of number talks as students who feel their responses are unappreciated by other students may be reluctant to participate; (c) the role of the classroom teacher is vital to the success of number talks; and (d) the dialogue during number talks was dominated by student-to-teacher discourse. Opportunities for future research include: (a) investigating the reliance on mental strategies learned through classroom number talks; (b) exploring the effects of number talks in a small-group setting; (c) examining the number talks practices of teachers at different grade levels; and (d) studying the impact of classroom number talks on students in various subgroups.
CHAPTER 1

INTRODUCTION TO THE STUDY

Students in the United States consistently perform lower than students in other industrialized nations on international assessments, particularly in the areas of mathematics and science (National Commission on Excellence in Education [NCEE], 1983; Spring, 2011). The Trends in International Mathematics and Science Study (TIMSS) showed that, compared to countries and benchmarking entities that participated in the 2015 TIMSS mathematics assessment, fourth-grade students in the United States ranked 10th in overall mathematics achievement (National Center for Education Statistics [NCES], 2016). Furthermore, results of the 2017 National Assessment of Educational Progress (NAEP) indicated that only 40% of fourth-grade students in the United States performed at or above the proficient level in mathematics (NAEP, 2018). Score comparisons between the 2015 and 2017 administrations of the NAEP mathematics assessment revealed no significant change in achievement for the percentage of fourth-grade students at or above the proficient level (NAEP, 2018). The poor performance of U.S. students in mathematics is attributable to colossal achievement gaps in learning and understanding and a lack of rigorous instruction and tasks that require extensive thinking and reasoning (Burns, 2012; Klein, 2003; Parrish, 2014). Without a shift in classroom practices to nurture proficiency in mathematics, U.S. students will continue to shadow their global peers, threatening the potential for international competition.
Since the Soviet Union’s launch of Sputnik in 1957, U.S. schools have repeatedly been labeled as failing (Spring, 2011). Politicians and public leaders alike blamed public schools for the Soviet Union’s launch of a space satellite before the United States, consequently holding schools accountable for inadequately preparing students to be globally competitive. As a result, the apparent lag of the United States behind the Soviet Union in the space race resulted in a national campaign to strengthen the math and science curricula in an effort to better prepare U.S. students to compete in the pursuit of technological and military advances (Spring, 2011). In addition to increased curricula expectations, the federal government established programs to improve U.S. schools by funding initiatives that addressed issues of poverty and academic failure (Spring, 2011).

As federal policymakers regulated educational reform efforts from the nation's capital, the NCEE captured the attention of the American people in 1983 when it released its report, *A Nation at Risk*. In addressing the nation, the commission focused on a diverse range of issues and shortcomings regarding U.S. education and cautioned, “Our nation is at risk [for] the educational foundations of our society are presently eroded by a rising tide of mediocrity that threatens our very future as a Nation and a people” (NCEE, 1983, p. 7). Even more compelling was the commission’s statement that “if an unfriendly foreign power had attempted to impose on America the mediocre educational performance that exists today, we might well have viewed it as an act of war” (NCEE, 1983, p. 7). Unlike other education reform efforts and reports, *A Nation at Risk* (1983) created widespread public concern about the future of education in the United States (Klein, 2003). The NCEE (1983) criticized public schools for focusing on rudimentary
skills in reading and computation while placing little emphasis on higher-order intellectual skills, such as comprehension, analysis, and problem solving. Moreover, the NCEE (1983) revealed that scores of U.S. students on international assessments were deficient when compared to students in other industrialized nations.

In response to accusations by the NCEE (1983) surrounding deficiencies in the mathematics curriculum in U.S. education, the National Council of Teachers of Mathematics (NCTM) developed the *Curriculum and Evaluation Standards for School Mathematics* (1989). Developers of these standards called for radical change in the conceptualization of mathematics and supported progressive instructional methods that advocated for exploratory and student-centered learning in education (Klein, 2003). NCTM (1989) recommended the placement of more attention on understanding number sense, estimation, and reasoning, with less emphasis on basic computation, rote practice, and memorization of skills for students in kindergarten through fourth grade. According to NCTM’s (1989) standards, a reformed mathematics classroom requires students to learn mathematics through “investigating, formulating, representing, reasoning, and using appropriate strategies to solve problems, and then reflecting on how mathematics is being used” (Romberg, 1993, p. 37). This call to action in mathematics reform affirmed the need for student access to more rigorous cognitive tasks to develop authentic reasoning and understanding in mathematics.

While NCTM’s (1989) standards faced opposition by many in the late 20th century, they paved the way for standards-based reform in curriculum, teaching, and assessment in mathematics (NCTM, 2000). In 2000, NCTM released the *Principles and
Standards for School Mathematics, which introduced interconnected content and process standards that spanned across all grade levels, prekindergarten through 12. Standards, as defined by NCTM (2000), are “descriptions of what mathematics instruction should enable students to know and do” (p. 7). Content standards are the five mathematical content strands that students are expected to learn at each grade band. These strands include number and operations, algebra, geometry, measurement, and data analysis and probability (NCTM, 2000). Process standards address the methods for attaining and applying content knowledge. The processes consist of problem solving, reasoning and proof, connections, communication, and representation (NCTM, 2000). Essentially, NCTM (2000) maintained that excellence in mathematics entails (a) high expectations and support for all students, (b) coherent and focused curricula that are clearly communicated across grades, (c) knowledge of the strengths and weaknesses in student learning, (d) construction of new understanding developed from previous student experiences and background knowledge, (e) assessment usage to guide teaching and support learning, and (f) integration of technology.

In 2001, the National Research Council (NRC) released Adding It Up: Helping Children Learn Mathematics. NRC members echoed other critics’ of U.S. mathematics education declarations that few students had the knowledge, skill, or confidence needed to apply learned mathematics. Council members coined the term mathematical proficiency to describe the five interrelated strands necessary to foster an individual’s knowledge and learning of mathematics. The NRC (2001) defined mathematical proficiency as the ability to demonstrate knowledge of mathematics through:
• conceptual understanding—comprehension of mathematical concepts, operations, and relations;
• procedural fluency—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
• strategic competence—ability to formulate, represent, and solve mathematical problems;
• adaptive reasoning—capacity for logical thought, reflection, explanation, and justification; and
• productive disposition—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

(p. 116)

The NRC (2001) suggested that daily mathematics instruction should incorporate each of the components as “the five strands are interwoven and interdependent in the development of proficiency in mathematics” (p. 116). Contrarily, mathematics teachers in the United States tend to emphasize some of the components during instruction while excluding others. Too often, teaching methods center on rote learning that accentuate procedures followed by repeated practice (NRC, 2001).

Common Core State Standards Initiative

Although teachers of mathematics in the United States have primarily focused on procedural knowledge (Burns, 2012; Klein, 2003; NCTM, 1989; NRC, 2001), it is equally important to develop mathematical understanding in students (Common Core
As stated in the *Common Core State Standards for Mathematics* (CCSSI, 2018a):

One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as \((a + b)(x + y)\) and a student who can explain where the mnemonic comes from. The student who can explain the rule understands the mathematics. (p. 4)

Prior to the establishment of the Common Core State Standards (CCSS), individual states created and adopted academic standards that identified the expectations of student learning at each grade level. Furthermore, each state also defined the level of academic proficiency that students were required to achieve (CCSSI, 2018b). States varied in their learning expectations for students, and these differences amounted to inconsistencies in student knowledge upon exiting high school. Throughout the nation, prospective high-school graduates were meeting proficiency levels on state graduation assessments, but many students needed remedial courses when entering schools of higher education. The differences in academic standards among states influenced a state-led initiative to develop the CCSS (CCSSI, 2018b).

In 2009, the National Governor’s Association Center for Best Practices (NGA) and the Council of Chief State School Officers (CCSSO) launched the development of the CCSS in an effort to establish a set of shared goals and expectations that students
should know and understand upon completion of a grade level (CCSSI, 2018b, 2018d). In developing the CCSS, state school chiefs and governors across the nation recognized the value of standards that were consistent between states, included real-world learning goals, and ensured that all students, regardless of geographical location, were college- and career-ready following high-school graduation (CCSSI, 2018b). Leaders of the initiative incorporated critical-thinking, problem-solving, and analytical skills into the standards, as these areas were deemed a necessity in preparing students for college and career (CCSSI, 2018e). Developers of the CCSS relied on already existing state standards, international benchmark standards, teacher experience, content experts, and public feedback to inform their decisions (CCSSI, 2018b). The NGA and CCSSO released the CCSS in June 2010 along with a validation report stating that the standards were (a) aligned with the knowledge and skills necessary for students to be college- and career-ready, (b) clear and specific in defining student expectations, (c) comparable to the standards of other top-performing nations, (d) research- and evidence-based, (e) reflective of best practices for standards development, (f) solid for cross-state adoption, and (g) a foundation for future development of quality assessments (CCSSI, 2018b).

The CCSS for Mathematics (CCSSM) consist of a two-part structure: Standards for Mathematical Practice (practice standards) and Standards for Mathematical Content (content standards) (Burns, 2012). Both the practice standards and the content standards reflect the skills and content knowledge necessary for students to be successful in college, career, and life (Burns, 2012; CCSSI, 2018a). The practice standards are the same across each grade band, kindergarten through twelfth, and guide teachers in the daily practices
that students should actively engage in when learning mathematics (Burns, 2012). As stated by leaders of the CCSS initiative, “The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students” (CCSSI, 2018a, p. 6). CCSS developers recognized the value of NCTM’s (2000) process standards and the NRC’s (2001) components for mathematical proficiency in creating the Standards for Mathematical Practice (CCSSI, 2018a). Elements of NCTM’s (2000) process standards, which include problem solving, reasoning and proof, connections, communication, and representation, are evident throughout the practice standards. Additionally, components of the NRC’s (2001) proficiency strands (i.e., conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition) emerge throughout the standards (CCSSI, 2018a).

While the Standards for Mathematical Practice direct teachers in the daily practices that students should engage in during instruction, the Standards for Mathematical Content “define what students should understand and be able to do in their study of mathematics” (CCSSI, 2018a, p. 4). In developing the CCSSM, leaders of the initiative recognized the issues associated with “a mile-wide, inch-deep curriculum” (CCSSI, 2018c, para 2). Decades of research studies in mathematics education found that educational institutions of top-performing countries relied on standards substantially more focused and coherent than the standards utilized by schools in the United States (CCSSI, 2018c). As a result, three major shifts were evident in the content standards.
First, the CCSSM cover fewer topics per grade level, which allows students to gain a more in-depth understanding of the standards introduced (CCSSI, 2018c). Furthermore, clarity and specificity within the standards give students the foundational skills needed to build solid understanding in mathematics. The standards prepare students to think and reason mathematically when applying concepts to new situations (CCSSI, 2018c).

Second, the CCSSM rely on coherence across grade levels (CCSSI, 2018c). The standards consist of skills and concepts that are interconnected from grade to grade with logical progressions that help students construct meaning from their previous learning experiences. Instead of learning standards in isolation, students apply and extend their knowledge of prior concepts to make sense of new content and to solve problems (CCSSI, 2018c).

Third, the CCSSM increase the level of rigor expected of students. According to the NGA and CCSSO, “Rigor refers to deep, authentic command of mathematical concepts, not making math harder or introducing topics at earlier grades” (CCSSI, 2018c, para. 5). The NGA and CCSSO identified three aspects of rigor that must be equally acknowledged for students to be mathematically sound: application, conceptual understanding, and procedural skills and fluency (CCSSI, 2018c).

Conceptual Framework

The conceptual framework for this study was classroom number talks and the role of this instructional practice in developing number sense and number relationships in third-grade students. Parrish (2014) defined number talks as five- to fifteen-minute
“classroom conversations and discussions around purposefully crafted computation problems” designed to “elicit specific strategies that focus on number relationships and number theory” (p. 5). The consistent implementation of number talks fosters the development of computational fluency and promotes sense making in mathematics by involving students in classroom discourse that leads to accurate, efficient, and flexible computation strategies. Parrish (2011, 2014) identified five fundamental components for effective implementation of a classroom number talk: (a) a classroom environment that is safe for the sharing of student ideas, (b) a facilitator that questions instead of telling students, (c) mental math strategies that encourage efficient strategies, (d) purposefully selected computation problems that guide students in developing mathematical relationships and patterns, and (e) discourse that is rich in communicating mathematical knowledge. Table 1 displays Parrish’s five tenets and corresponding characteristics.

Table 1

*Five Tenets of Number Talks*

<table>
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<th>Tenet</th>
<th>Characteristics</th>
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| Classroom environment and community  | • Ideas accepted without judgment  
                                         | • Respectful, risk-free environment                                              |
| Teacher’s role                       | • Facilitator  
                                         | • Questioner  
                                         | • Listener  
                                         | • Learner                                                  |
| Role of mental math                  | • Develops number relationships that lead to mathematical fluency               |
| Purposeful computation problems      | • Intentional problems focused on mathematical relationships                   |
| Classroom discussions                | • Explain and investigate number concepts  
                                         | • Acquire a toolbox of efficient and flexible strategies                         |
During a classroom number talk, students are introduced to a string of closely related computation problems, called number strings, which consist of a series of computation problems purposefully constructed to demonstrate mathematical patterns and relationships between numbers (Parrish, 2011, 2014). After forming a mental strategy or strategies for solving a problem in the number string, students convey to the classroom teacher through discreet hand signals they have one or more methods for solving the problem. Once students have engaged with the problem for a sufficient amount of time, the teacher facilitates a whole-group discussion where students share and justify their approaches for mentally calculating the solution through verbal reasoning with their peers. This routine allows students to challenge their own thinking, as well as the reasoning of others, while making connections between numerical relationships and diverse strategies that lead to mathematical understanding. As a result, students are able to augment their toolbox of efficient and flexible strategies for solving computation problems mentally and fluently (Parrish, 2011, 2014).

Theoretical Framework

The theoretical underpinnings of this research study were grounded in the theory of constructivism. Brooks and Brooks (2001) defined constructivism as “a theory of learning that places the quest for understanding at the center of the educational enterprise” (p. 150). Although theorists offer different viewpoints for constructivist learning, Noddings (1990) identified four, generally accepted positions:

1. All knowledge is constructed. Mathematical knowledge is constructed, at least in part, through a process of reflective abstraction.
2. There exist cognitive structures that are activated in the processes of construction. These structures account for the construction; that is, they explain the result of cognitive activity in roughly the way a computer program accounts for the output of a computer.

3. Cognitive structures are under continual development. Purposive activity induces transformation of existing structures. The environment presses the organism to adapt.

4. Acknowledgement of constructivism as a cognitive position leads to the adoption of methodological constructivism.
   a. Methodological constructivism in research develops methods of study consonant with the assumption of cognitive constructivism.
   b. Pedagogical constructivism suggests methods of teaching consonant with cognitive constructivism. (p. 10)

Constructivist practices can either be cognitive or social in nature, but both forms of constructivism are necessary for effective classroom instruction (Powell & Kalina, 2009). Cognitive constructivism is rooted in the work of Jean Piaget, a Swiss epistemologist who studied the source of knowledge in children (Kamii, 2000). Piaget focused on the construction of knowledge through one’s own personal experiences (Powell & Kalina, 2009). Social constructivism is the product of the work of Vygotsky’s, a Soviet psychologist who believed social interaction, language, and cooperative learning were integral parts of the learning process (Powell & Kalina, 2009).
Together, cognitive and social constructivism form the foundation for constructivist learning in mathematics.

Statement of the Problem

Mathematics education in the United States has traditionally focused on repetition and memorization of rudimentary skills and procedures (Burns, 2012; Klein, 2003; NCTM, 1989; NRC, 2001; Parrish, 2014; Sood & Mackey, 2014). The National Education Association (NEA) (2012) reported,

America’s system of education was built for an economy and a society that no longer exists. In the manufacturing and agrarian economies that existed 50 years ago, it was enough to master the “Three Rs” (reading, writing, and arithmetic). In the modern “flat world,” the “Three Rs” simply aren’t enough. If today’s students want to compete in this global society, however, they must also be proficient communicators, creators, critical thinkers, and collaborators (the “Four Cs”). (p. 5)

Although communication is a fundamental component of mathematics instruction (NCTM, 2000), the teacher’s role in U.S. classrooms has historically centered on telling with few opportunities for students to collaboratively think and reason through problems (Lampert, 1990). Compared to students in Asian classrooms, U.S. students are far less involved in conversations about mathematics (Sims, 2008). International assessment scores consistently show that students in developed countries outperform U.S. students in mathematics (NCES, 2016; Spring, 2011). Furthermore, national assessments indicate that less than half of U.S. students are proficient in mathematics (NAEP, 2018).
Researchers widely acknowledge that discourse is essential to developing mathematical reasoning and understanding (Franke et al., 2009; Lampert, 1990; NCTM, 2000). However, classroom practices continue to reflect teacher-directed instruction with minimal student engagement.

Number sense is at the core of mathematics understanding (Boaler, 2015; NCTM, 2000; Washington, 2015). Student exposure to intentional tasks that promote number sense and reasoning are critical to developing mathematical proficiency. Unfortunately, unbalanced instructional practices that emphasize computation procedures and algorithms considerably outweigh the amount of time devoted to efficient and flexible strategies that cultivate number sense and reasoning (Burns, 2012). By design, number talks foster number sense and reasoning in students (Parrish, 2014). As stated by Parrish (2014), “Every time you elicit answers to a problem in a number talk and ask students to share whether the proposed solutions are reasonable, you are helping to build number sense” (p. 36). Establishing an environment that allows students to flexibly grapple with number relationships, particularly through mental processes, supports students in reasoning about mathematical concepts (Van de Walle, Karp, & Bay-Williams, 2013). Unbalanced mathematics instruction that fails to strengthen understanding is problematic for students, thus prompting an unrelenting fear of mathematics.

Purpose of the Study

Advocates for quality mathematics instruction contend that students across all grade levels, kindergarten through 12, must articulate reasoning and justification of problems to generate understanding. Both NCTM (2000) and the NRC (2001) called for
increased integration of classroom practices such as reflecting, reasoning, and explaining to generate mathematical thinking in students when solving problems. In creating the CCSS, members of the NGA and CCSSO (CCSSI, 2018a) incorporated the Standards for Mathematical Practice, along with the Standards for Mathematical Content, to improve teaching and learning in mathematics. A review of the literature indicated the importance of strengthening number sense, reasoning, and understanding in mathematics students (Boaler, 2015; NCTM, 2000; NRC, 2001). However, a lack of existing empirical research failed to show how classroom number talks support third-grade students’ reasoning and understanding of number concepts in mathematics pursuant to teacher perception.

Research Questions

The guiding question for this qualitative study was

1. What is the role of number talks in developing number sense and number relationships in third-grade students pursuant to teacher perception?

The following subquestions supported the guiding question:

a. How do teachers develop classroom community in number talks?

b. How do teachers perceive their role in classroom number talks?

c. How do mental computation strategies support students with mathematical understanding as observed by teachers?

d. How do teachers select purposeful computation problems for classroom number talks?
e. How do number talks promote student discourse in mathematics as reported by teachers?

f. What questions do teachers ask to elicit student responses during number talks?

Limitations

Each number talk observation in this study occurred in a whole-group setting, making it difficult to monitor the mathematical discourse between students. Parrish (2014) maintained, “Incorporating small-group number talks throughout the week provides greater opportunities to focus on individual students” (p. 26). Furthermore, Parrish (2014) claimed that “forming small, intimate groups removes the veil of anonymity and allows you to focus more carefully on individual strengths as well as factors interfering with understanding” (p. 26). Although teachers encourage students to share their thinking with the entire class, it is not feasible for all individuals to receive personalized attention in a large group. Additionally, some students may be reluctant to communicate their reasoning in a whole-group setting. A small-group setting can provide a comfortable atmosphere for students to develop the confidence needed to convey their ideas.

Due to time constraints, this research study took place over a six-week period. While the purpose of this study was to establish the role of number talks in developing number sense, number relationships, and mental computation in third-grade students, a year-long study would allow the researcher to observe the progression of both teacher and student comfort levels with the practice. The culture of classroom learning
communities evolves over time, supporting productive discourse that promotes meaningful contributions to the discussion. A longer study period could acknowledge the shifts in dialogue as the teacher nurtures student reasoning through precise questioning.

The participants for this case study were selected because of convenience and accessibility to the researcher. However, each case met the criteria for participation in the study since (a) each participant was assigned to teach third-grade mathematics during the 2018-2019 school year at one of the three settings identified for inclusion in this study; (b) each participant had a minimum of five years teaching experience; (c) each participant had completed a K-5 mathematics endorsement program; (d) each participant was proficient in facilitating classroom number talks; and (e) each participant consistently facilitated classroom number talks as part of instruction in mathematics.

Assumptions

In conducting this study, I assumed that all participants were comfortable with contributing to the study and were candid in their responses to the interview questions. Additionally, I assumed that video and audio recording did not influence the behaviors of any participants because I established appropriate measures to ensure confidentiality and anonymity. Since I was not responsible for evaluating any of the participants, teachers or students, there were not any conflicts of interest for participation.

The teachers selected for participation in this study worked at different schools with diverse student populations and various socioeconomic needs. This was critical to the study since I assumed these geographical differences provided a broader perspective regarding the impact of number talks in developing students’ number sense, number
relationships, and mental computation strategies. Consequently, I assumed the results of the study were transferable to other settings.

One of the criteria for teacher inclusion in this study was an endorsement in K-5 mathematics. It was assumed that teachers with an endorsement in mathematics were confident and knowledgeable of the mathematics curriculum for their grade level.

Definitions of Key Terms

The following terms were essential to this study, for they directly relate to the learning and understanding of mathematics:

Accuracy denotes solving problems precisely. Students who solve problems accurately work meticulously to ensure proper arithmetic and recording of their work, and they use reasoning skills to determine if their answers are logical (Russell, 2000).

Algorithms are conventional rules or a series of procedures for solving a computation problem (Ashlock, 2006; Kamii & Dominick, 1998). In this study, algorithms refer to a set of rote procedures that typically cannot be solved in a mental capacity.

Computational fluency is “having efficient and accurate methods for computing” (NCTM, 2000, p. 152).

Conceptual understanding is the “comprehension of mathematical concepts, operations, and relations” (NRC, 2001, p. 5).

Constructivism is “a theory of learning that places the quest for understanding at the center of the educational enterprise” (Brooks & Brooks, 2001, p. 150).
Discourse “refers to the ways of representing, thinking, talking, and agreeing and disagreeing that teachers and students use to engage in . . . tasks” (NCTM, 1991, p. 4).

Efficiency refers to a student’s ability to solve problems easily using logic or sense making. Students who are efficient in mathematics are able to execute appropriate strategies properly to find a solution (Russell, 2000).

Flexibility involves selecting an appropriate method to solve a problem. Students who demonstrate flexibility in mathematics are able to approach problems from multiple perspectives and use various strategies to solve their work (Russell, 2000).

Fluency describes an individual’s ability to solve problems efficiently, accurately, and flexibly (NCTM, 2000; Russell, 2000).

Math talk refers to the “explanations, declarations of formal principles or procedures, and other mathematical statements” (Sims, 2008, p. 121).

Mental computation is the ability to mentally “calculate exact numerical answers without the aid of any calculating or recording device” (Reys, 1985, p. 43).

Number sense denotes “a person’s general understanding of number and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgements and to develop useful strategies for handling numbers and operations” (McIntosh, Reys, & Reys, 1992, p. 3).

Number talks are five- to fifteen-minute “classroom conversations and discussions around purposefully crafted computation problems” designed to “elicit specific strategies that focus on number relationships and number theory” (Parrish, 2014, p. 5).
Representations are the “processes and products that are observable externally as well as to those that occur ‘internally,’ in the minds of people doing mathematics” (NCTM, 2000, p. 67). Examples of representations include diagrams, graphical displays, pictures, and symbolic expressions (NCTM, 2000).

Summary

Students in other industrialized nations consistently outperform students in the United States on international assessments in mathematics and science (NCEE, 1983; Spring, 2011). Furthermore, national assessment results indicate extensive achievement gaps in students’ knowledge of mathematics (NAEP, 2018). Reform efforts accentuate the importance of number sense and reasoning in mathematics (NCTM, 1989), yet the inclination of teachers across the nation is to focus on basic computation and procedural skills instead of conceptual understanding (Burns, 2012; Klein, 2003; NCTM, 1989; NRC, 2001; Parrish, 2014; Sood & Mackey, 2014). The development of the CCSSM reinforced the need for critical thinking, problem solving, and analytical skills in mathematics (CCSSI, 2018e) and identified both content and process standards that acknowledged the skills necessary for students to be college- and career-ready (Burns, 2012; CCSSI, 2018a). To support deeper learning, the CCSSM focused on fewer topics, presented coherence across grade levels, and increased the level of rigor in mathematics (CCSSI, 2018c). The intent of classroom number talks is to foster the essence of learning in mathematics by promoting number sense understanding, computational fluency, mental computation strategies, and flexible thinking and reasoning (Parrish, 2014).
The purpose of this study was to investigate the role of number talks in developing number sense and number relationships in third-grade students pursuant to teacher perception. This research study also explored the instructional practices of teachers as defined by Parrish’s (2011, 2014) five tenets of number talks, along with the types of questions that teachers ask to elicit student responses during number talks.

This chapter introduced the study and provided an overview of the conceptual and theoretical frameworks, the research questions, a summary of the purpose, and definitions of key terms relevant to the study. Chapter 2 presents a review of the relevant literature associated with the topic. Chapter 3 offers a description and rationale of the proposed research design and methodology. Chapter 4 provides an overview of the findings. Chapter 5 discusses the conclusions, implications, and recommendations of the study.
CHAPTER 2
REVIEW OF THE LITERATURE

Historically, mathematics instruction in the United States has centered on rote practices that highlight procedural knowledge and basic computation (Burns, 2012; Klein, 2003; National Council of Teachers of Mathematics [NCTM], 1989; National Research Council [NRC], 2001; Parrish, 2014; Sood & Mackey, 2014). In many mathematics classrooms, the emphasis on developing mental math strategies receives far less attention when compared to the amount of time devoted to paper-and-pencil computation practice (Burns, 2012). While teaching computation skills are an important part of a balanced curriculum, relying on procedures alone does not allow students to investigate problems using the level of in-depth thinking that the standards for mathematical practice require (Burns, 2012), since the practice standards promote instruction in mathematics based on conceptual understanding and mathematical reasoning (Common Core State Standards Initiative [CCSSI], 2018). Van de Walle, Karp, and Bay-Williams (2013) claimed, “Flexible methods for computing, especially mental methods, allow students to reason much more effectively in every area of mathematics involving numbers” (p. 216). The issue with instructional practices in mathematics classrooms across the nation is the lack of attention placed on reasoning and understanding in mathematics through the development of mental strategies.
The purpose of this study was to investigate the role of classroom number talks in developing number sense and number relationships in elementary mathematics students. The intent of this chapter is to provide a comprehensive review of the literature comprised of the most prevalent research studies associated with the topic. The first section of the chapter examines the historical and theoretical foundations that influenced the study, followed by a review of number sense, mental computation, and discourse—the principal objectives for classroom number talks. The remainder of the chapter reviews the empirical research studies related to number talks.

Theoretical Framework: Constructivism

*Mathematical learning is not a process of internalizing carefully packaged knowledge but is instead a matter of reorganizing activity, where activity is interpreted broadly to include conceptual activity or thought* (Cobb et al., 1991, p. 5).

As defined by Brooks and Brooks (2001), constructivism is “a theory of learning that places the quest for understanding at the center of the educational enterprise” (p. 150). Constructivism is experiential learning through real-life experiences, which are used to construct meaning and knowledge; it is based on a relativist ontology, meaning that knowledge is a social reality, and individuals construct their own reality through multiple interpretations (Guba & Lincoln, 1989). This theoretical assumption elicits that students are sense-makers who construct their own knowledge as active participants in the learning process, resulting in independent and autonomous thinkers. In other words, “humans *create* meaning as opposed to *acquiring* it” (Ertmer & Newby, 1993, p. 62). Because students receive the autonomy and initiative to challenge conjectures, they seek ways to build connections between ideas and concepts by formulating questions and
issues and analyzing the situation (Brooks & Brooks, 2001). Accordingly, students become problem solvers, and perhaps even problem finders, who are accountable for their learning (Brooks & Brooks, 2001).

The constructivist paradigm insinuates that learners “come to formal education with a range of prior knowledge, skills, beliefs, and concepts that significantly influence what they notice about the environment and how they organize and interpret it” (Bransford, Brown, & Cocking, 2000, p. 10). Consequently, this impacts how individuals retain information, reason through situations, problem solve, and obtain new knowledge (Bransford et al., 2000). Since individuals construct new knowledge from preexisting knowledge, it is critical to investigate and dissect student misconceptions. Students with preconceived notions need opportunities to engage in rigorous activities that challenge their beliefs and cultivate an accurate understanding of the content. By alleviating misconstructions early in learning, individuals are able to avoid the confusion and frustration that is likely to occur from unlearning and relearning information (Bransford et al., 2000). Explicitly, teachers must know and understand where students are in their learning to design and plan appropriate instruction for the learner (Greenes, 2008).

Classroom instruction rooted in constructivist teaching practices emphasizes the significance of the active learner and relies on the student’s ability to expand upon and interpret information (Duffy & Jonassen, 1991). It is important to note that constructivism is not synonymous with discovery learning; indeed, constructivism can
assume the role of direct instruction provided students go beyond the information communicated by the teacher (Duffy & Jonassen, 1991).

Unlike the theory of constructivism, the transmission view of teaching characterizes learning as a transfer of knowledge from the teacher to the student through words and actions or by objects in the environment (Cobb, 1988). Cobb (1988) reported, “An abundance of research indicates that students routinely use prescribed methods to solve particular sets of tasks on which they have received instruction without having developed the desired conceptual understanding” (p. 90). As a result, the transmission of information from teacher to student is only presumed effective if the instructional goal is for students to acquire a specific skill set or regurgitate rote methods when solving problems (Cobb, 1988).

Conversely, the constructivist perspective supports the development of conceptual structures that bestow the knowledge students need to persevere in problem solving, to apply skills to a variety of mathematical situations, and to make connections to other content domains; the structures of knowledge become a permanent fixture in the student's problem-solving toolbox (Cobb, 1988). Ultimately, the goal of instruction is to build upon the existing knowledge of students and to teach for understanding (Greenes, 2008). These two pedagogical differences in instructional delivery have profound implications on how students choose to approach problems in mathematics.

The instructional framework in a constructivist environment shifts from the traditional classroom model of teacher-directed lessons, which involves the passive transfer of knowledge from teacher to student, to a student-centered learning approach,
where students are actively engaged participants in the problem-solving process (Ertmer & Newby, 1993). The teacher plays a critical role in planning and structuring lessons that introduce problems at varying degrees of complexity and encourage collaboration among students (Ertmer & Newby, 1993; Greenes, 2008; Yackel, Cobb, Wood, & Merkel, 1990). Because instruction embedded in the practices of constructivism does not rely on the transmission of isolated facts and repeated practice of basic skills, the challenge for the teacher is to design instruction that is “problematic for children” (Yackel, Cobb, & Wood, 1991, p. 396) and promotes conceptual understanding of mathematics. Consequently, the teacher must have a deep and multifaceted knowledge of mathematics to lead and facilitate rigorous instruction that develops conceptual understanding for students (Cobb, 1988; Greenes, 2008). Yackel and colleagues (1991) claimed, “Conceptually based mathematics learning, as indicated by the ability to solve problems in a wide variety of situations, is precisely the type of learning that requires disequilibrium, conflict, and problem solving” (p. 396). Precisely, teachers nurture productive struggle in mathematics by engaging students in tasks that spark their interest, posing questions that entice curiosity, assessing progress, providing meaningful feedback, and encouraging quality discourse among students (Greenes, 2008).

Communication is an essential component of the balanced mathematics classroom (NCTM, 2000). Both teacher-to-student and student-to-student interactions should transpire as part of daily instruction. Effective collaboration and discourse between students involve more than a review and comparison of solutions; they require explaining personal thought process in a meaningful way to another individual and deciphering the
interpretations of others (Yackel, Cobb, Wood, & Merkel, 1990; Yackel, Cobb, Wood, Wheatley, & Merkel, 1990). Put simply, the student’s role in discourse is to “listen and reflect on what is being said and try to make sense of it in terms of their own cognitive framework” (Yackel, Cobb, Wood, Wheatley, & Merkel, 1990, p. 18). The teacher’s role is to establish an environment favorable for discourse and to facilitate the discussion by posing questions. As a facilitator, the teacher is responsible for supporting students to articulate their thinking as well as encouraging conceptualization of the topic using inventive and unconventional methods (Yackel, Cobb, Wood, & Merkel, 1990; Yackel, Cobb, Wood, Wheatley, & Merkel, 1990). Foremost, it is not the duty of the facilitator to impart knowledge of right or wrong answers to students, nor to guide students in finding a correct solution (Yackel, Cobb, Wood, Wheatley, & Merkel, 1990). The expectation is that students work cooperatively with peers and engage in conversations that verbalize thinking, validate reasoning, and invite clarity of misconstrued information (Yackel, Cobb, Wood, Wheatley, & Merkel, 1990).

Constructivism is a theory or learning that harnesses the relationship between the learner and the environment to construct knowledge (Ertmer & Newby, 1993). Essentially, the learner creates new meaning by applying prior knowledge and experiences to authentic tasks in meaningful contexts (Ertmer & Newby, 1993). Teachers who adopt constructivist practices embrace students as active participants in the learning process, and they support students in elaborating upon and interpreting information (Duffy & Jonassen, 1991). Foremost, the constructivist mathematics teacher must be knowledgeable of the subject matter (Cobb, 1988; Greenes, 2008), since he or
she is responsible for developing rigorous tasks, cultivating understanding through problem-posing and questioning, facilitating effective classroom discourse, and providing meaningful feedback (Greenes, 2008).

Historical and Philosophical Underpinnings of Constructivism

For centuries, philosophers have speculated about the phenomena of knowledge. As early as sixth century B.C.E., Xenophanes of Colophon, a western philosopher whose epistemological skepticism challenged traditional principles, acknowledged a distinction between knowledge and belief in that “God knows the truth, but belief is allotted to men” (Xenophanes as cited in Lesher, 1978, p. 3). The implications for this assumption suggest that knowledge evolves from experience, and the inconceivable notion of truth as an unattainable reality (von Glasersfeld, 1990). To know truth, “we would need an access to such a world that does not involve our experiencing it” (von Glasersfeld, 1990, p. 20), and “whatever truth is to be gained must come as a result of human initiative and inquiry” (Lesher, 1978, p. 16). Xenophanes’s skeptic views revolutionized the way preSocratic philosophers contemplated the state of knowing. By the end of the fifth century B.C.E., the perplexing wonder of how one comes to know the world troubled the preSocratics, who sought to understand the truth of knowledge (von Glasersfeld, 1983).

The emphasis on critical thinking and creating meaning can be traced to fifth century B.C.E. when Socrates (468-399 B.C.E.), a Greek philosopher during the Classical Era, introduced his dialectical process of inquiry to the Athenians (Gutek, 2011). A critical thinker interested in discovering the meaning of life, Socrates initiated questions in ancient Greece that caused individuals to critically examine their beliefs and opinions.
Unlike the Sophists of Ancient Greece, who educated students on the knowledge and skills necessary for service in the democratic city-state (Manus, 1996), Socrates did not believe that knowledge could be transmitted from teacher to student (Gutek, 2011). According to Manus (1996), “Socrates did not transmit his beliefs but guided students to formulate their own, while the Sophists ‘told’ students what to believe. Thus, the students of the Sophists were receptors; whereas the students of Socrates were actors in the process” (p. 316).

The pedagogical differences between Socrates and the Sophists are essentially the same oppositions that continue to haunt mathematics classrooms across the United States. In 21st-century education, procedural methods dominate instructional practices as teachers inform students of the what and how of behavioral objectives (Manus, 1996). Much like the Sophists, the procedural teacher disseminates knowledge to students, expects compliance, determines techniques, and ignores self-reflection (Manus 1996). Contrarily, the constructivist teacher, like Socrates, poses questions, encourages inquiry and reflection, and fosters sense making and understanding through associations and relationships. Socrates perceived knowledge as a personal experience that examines the why of education (Manus, 1996). Gutek (2011) stated, “The teacher’s task, Socrates said, is to draw ideas out of students’ minds by asking them probing and challenging questions that cause them to think critically, deeply, and reflectively about their beliefs” (p. 35). Philosophically, Socrates believed that knowledge was a continuous cycle of honest debate that transpired through the process of inquiry and investigation of conjectures against reason and fact in the search for meaning (Cookson, 2009).
Approximately 2,000 years later during the Age of Enlightenment, an 18th-century Italian philosopher named Giambattista Vico (1668-1744) diverted from the traditional beliefs of ancient philosophers and proposed that “the human mind can know only what the human mind has made” (von Glasersfeld, 1990; 1992/2007). This radical supposition implied that individuals are not passive recipients of information; instead, they are active makers of knowledge (von Glasersfeld, 1992/2007). An epistemological pioneer of constructivism, Vico coined the phrase *verum est ipsum factum* (the true itself is made), meaning that knowledge is produced and constructed (von Glasersfeld, 1989). Vico’s philosophical views were mirrored by the beliefs of another prominent 18th-century philosopher named Immanuel Kant and later by a 20th century Swiss epistemologist named Jean Piaget.

Immanuel Kant (1724-1804), a German philosopher, claimed the conception of knowledge resulted from categories or mental structures in the mind, such as space, time, and causality, that exhibit a priori elements (Fabricius, 1983; Gutek, 2011). Kant’s rationale of space and time as a priori elements distinguishes him from Vico, who assumed that space and time were constructs of the human mind (von Glasersfeld, 1990). Unlike the empiricists, who believed that truth could only be derived from sense experience, Kant believed the construction of reality was a product of the relationship between mind and experience (Fabricius, 1983). As Kant (1781/1965) stated in his *Critique of Pure Reason*, “Thoughts without content are empty, intuitions without concept are blind . . . only through their union can knowledge arise” (p. 93). Kant’s revelation indicated a need to amalgamate rationalism, the theory that knowledge is
obtained through reason rather than experience, with empiricism (Fabricius, 1983). The marriage of rationalism and empiricism conceived the notion of constructivism.

Three Types of Knowledge

Piaget believed that children rely on their present knowledge and experience of the environment to construct reality and make sense of the world (Schunk, 2012). Although most commonly recognized for uncovering the four stages of cognitive development, sensorimotor, preoperational, concrete operational, and formal operational, the constructs of this study will focus on Piaget’s three types of knowledge: physical knowledge, social knowledge, and logico-mathematical knowledge.

Piaget (1970) defined knowledge as “a system of transformations that become progressively adequate” (p. 15). Physical knowledge is a concrete form of knowledge grounded in personal experiences (Piaget, 1970). Kamii (2000) portrayed physical knowledge as the “knowledge of objects in external reality” (p. 5). Properties of objects, such as color, weight, or size are examples of physical knowledge. Piaget (1970) stated,

In cases involving the physical world the abstraction is abstraction from the objects themselves. A child, for instance, can heft objects in his hands and realize that they have different weights—that usually big things weigh more than little ones, but that sometimes little things weigh more than big ones. All this he finds out experientially, and his knowledge is abstracted from the objects themselves. (p. 16)

Physical knowledge is largely attained through the external observation of objects in their natural environment (Kamii, 2000).
The second type of knowledge—social or conventional knowledge—is “knowledge born of experience” (Piaget, 1971, p. 100). Kamii and Dominick (1998) explained, “Whereas an ultimate source of physical knowledge is in objects, an ultimate source of social knowledge is in conventions made by people” (p. 132). Examples of social knowledge include the English language, names of holidays, customs, and traditional mathematical algorithms (Kamii, 1998, 2000).

The third type of knowledge, logico-mathematical knowledge, is the focal point of this study. Piaget (1971) defined logico-mathematical knowledge:

[a] kind of knowledge which is brought about by operational coordinations (functions, etc.) and corresponds, in biology, to regulation systems of any scale, in the hypothesis that elementary logical operations (revisions, dissociations, order, etc.), with their “necessary” characteristic of coherence or noncontradiction, represent the fundamental regulatory organ of intelligence. (p. 100)

In simpler terms, Kamii (2000) described logico-mathematical knowledge as an individual’s internal ability to construct mental relationships. Piaget (1970) presented the following example to explain logico-mathematical knowledge:

A small child . . . was counting pebbles one day; he lined them up in a row, counted them from left to right, and got ten. Then, just for fun, he counted them from right to left to see what number he would get, and was astonished that he got ten again. He put the pebbles in a circle and counted them, and once again there were ten. He went around the circle in the other way, and got ten again. And no matter how he put the pebbles down, when he counted them, the number came to
ten. He discovered here what is known in mathematics as commutativity, that is, the sum is independent of the order. (pp. 16-17)

In this scenario, the child ordered and lined the pebbles and discovered the sum through internal thought and actions (Piaget, 1970). The child determined that the order of the pebbles did not affect the sum, and as such, he formed a mental relationship based on the knowledge he constructed. While another individual could certainly explain this concept to the child, it “does not become the child’s knowledge until he or she makes the relationship” (Kamii & Dominick, 1998). Kamii and Dominick (1998) further asserted, A characteristic of logico-mathematical knowledge is that there is nothing arbitrary in it. For example, adding 356 to 278 results in 634 in every culture. The social (conventional) rule, or algorithm, stating that one must add the ones first, then the tens, and then the hundreds is arbitrary. The teaching of algorithms is based on the erroneous assumption that mathematics is a cultural heritage that must be transmitted to the next generation. . . . Children in the primary grades should be able to invent their own arithmetic without the instruction they are now receiving from textbooks and workbooks. (p. 132)

With logico-mathematical knowledge, children exercise knowledge of previous mental relationships to develop new relationships (Kamii, 2000). For example, when considering the relationship of three fives, a student concludes that $5 + 5 + 5$ is 15. As the student progresses in his or her knowledge and understanding of number and operations, the child connects that $5 + 5 + 5 = 15$ is the same as $3 \times 5 = 15$. In this instance, logico-mathematical knowledge is evident in the child’s construction of the
relationship between repeated addition and multiplication. Hence, the source of logico-mathematical knowledge is internally constructed in each child’s mind (Kamii, 1996).

Social Constructivism

While Piaget’s cognitive theory contends that learning is largely a constructivist process based on personal experiences, Vygotsky (1934/1978) accentuated the importance of a social environment that nurtures teacher-to-student and student-to-student interactions to construct knowledge (Powell & Kalina, 2009; Schunk, 2012). Vygotsky’s theory of learning, social constructivism, “acknowledges that both social processes and individual sense making have central and essential parts to play in the learning of mathematics” (Ernest, 1994, p. 63). Vygotsky believed that both language and cultural context were critical for meaning and understanding (Bruner, 1997) and social and interactive experiences that directly relate to an individual’s surroundings transform learning (Schunk, 2012). Vygotsky (1934/1978) claimed, “The most significant moment in the course of intellectual development, which gives birth to the purely human forms of practical and abstract intelligence, occurs when speech and practical activity, two previously completely independent lines of development, converge” (p. 24). In other words, social interaction between students in the classroom environment has a powerful effect on the internalization of knowledge and the depth of learning (Powell & Kalina, 2009).

Number Sense

*Number sense can be described as good intuition about numbers and their relationships. It develops gradually as a result of exploring numbers, visualizing them in*
a variety of contexts, and relating them in ways that are not limited by traditional algorithms (Howden, 1989, p. 11).

Number sense, a commonly referenced phrase in mathematics, is difficult to define because of the many constructs that frame the term (Greeno, 1991; McIntosh, Reys, & Reys, 1992; Okamoto, 2015; Sowder, 1992; Whitacre, Henning, & Atabaş, 2017; Yilmaz, 2017). Greeno (1991) proposed that number sense is comprised of interrelated properties that include flexible numerical computation, numerical estimation, and quantitative judgment and inference. Flexible numerical computation refers to the transformation of a problem using number equivalencies in order to mentally find a solution (Greeno, 1991). For example, to solve a problem such as 25 + 15, a student might decompose the addends to form a new problem: (20 + 5) + (10 + 5). Then, the corresponding place value positions are combined to find the partial sums of 30 + 10, which equals 40: (20 + 10) + (5 + 5). Numerical estimation involves the understanding of numerical values to determine the approximate magnitude of numbers (Greeno, 1991). For example, a student might reason that 224 x 4 is about 900 since 200 x 4 = 800 and 25 x 4 = 100. Quantitative judgment and inference involve making informed decisions about numerical values (Greeno, 1991). For instance, a student solves a division problem and finds a quotient of 12 with a remainder of 4. Problem context and understanding are necessary to determine the appropriateness of the remainder.

While mathematics educators may be inclined to develop specific activities intended to assist students with acquiring flexible numerical computation, numerical estimation, and quantitative judgment and inference, a comprehensive approach is to
recognize the development of number sense as the result of prolonged exposure to a broad collection of activities (Greeno, 1991). Textbook resources encourage number sense through intentionally crafted activities, but rarely do these resources promote number sense through a larger lens that encompasses relationships (Sowder, 1992). Howden (1989) claimed,

Textbooks are limited to paper-and-pencil orientation, they can only suggest ideas to be investigated, they cannot replace the “doing the mathematics” that is essential for the development of number sense. No substitute exists for a skillful teacher and an environment that fosters curiosity and exploration at all grade levels. (p. 11)

While it may be necessary to introduce activities intentionally designed to investigate number patterns and relationships, the emphasis should be placed on the students’ thinking about the connectedness of numbers and not on direct instruction of a skill (Cobb & Merkel, 1989; Greeno, 1991).

Number sense is at the heart of mathematical thinking, as it channels the selection, construction, and application of computational methods (McIntosh et al., 1992). McIntosh and colleagues (1992) defined number sense as “a person’s general understanding of number and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgements and to develop useful strategies for handling numbers and operations” (p. 3). After reviewing and analyzing themes that emerged in literature related to the subject, McIntosh et al. (1992) developed a framework to demonstrate the interdependence of the principal components of number
sense, in which they identified three distinct categories: “knowledge of and facility with numbers”, “knowledge of and facility with operations”, and “applying knowledge of and facility with number and operations to computational settings” (p. 4).

Sowder (1992) delineated nine dimensions or behaviors to denote the presence of number sense based on a compilation of previous research. According to Sowder (1992), number sense entails: (a) the “ability to compose and decompose numbers”, “to move flexibly among different representations”, and “to recognize when one representation is more useful than another”; (b) the “ability to recognize the relative magnitude of numbers” and the “abilities both to compare and to order numbers”; (c) the “ability to deal with the absolute magnitude of numbers”; (d) the “ability to use benchmarks”; (e) the “ability to link numeration, operation, and relation symbols in meaningful ways”; (f) “understanding the effects of operations on numbers”; (g) the “ability to perform mental computation through ‘invented’ strategies that take advantage of numerical and operational properties”; (h) “being able to use numbers flexibly to estimate numerical answers to computations, and to recognize when an estimate is appropriate”; and (i) “a disposition toward making sense of numbers” (pp. 5-6). In short, number sense is characterized as an individual’s understanding of numeration, number magnitude, mental computation, computational estimation, and disposition toward making sense of numbers (Sowder, 1992).

Although research extensively documents that number sense is difficult to define (Greeno, 1991; McIntosh et al., 1992; Okamoto, 2015; Sowder, 1992; Whitacre et al., 2017), Whitacre and colleagues (2017) claimed this is because of polysemy in the
literature. After a review of 124 research articles on the topic of number sense, Whitacre et al. (2017) found three different constructs associated with the term: innate number sense, early number sense, and mature number sense. The first construct, innate number sense, involves the instinctive knowledge of number and the discernment of number magnitudes. The early number sense construct involves the explicit learning of number concepts. The final construct, mature number sense, encompasses components of number sense (Greeno, 1991; McIntosh et al., 1992; Sowder, 1992) that focus on understanding rather than skills (Whitacre et al., 2017).

Developing Number Sense Understanding

In *Principles and Standards for School Mathematics* (NCTM, 2000), the authors acknowledged, “understanding number and operations, developing number sense, and gaining fluency in arithmetic computation form the core of mathematics education for the elementary grades” (p. 32). Number sense is the foundation for success and understanding in mathematics (Boaler, 2015; NCTM, 2000; Washington, 2015). Students who struggle in upper-level mathematics courses often flounder because of poor number sense (Boaler, 2015). Parrish (2014) asserted, “The teaching of mathematics has been viewed as a discrete set of rules and procedures to be implemented with speed and accuracy but without necessarily understanding the mathematical logic” (p. 4). Parrish (2014) further claimed, “Our classrooms are filled with students and adults who think of mathematics as rules and procedures to memorize without understanding the numerical relationships that provide the foundation for these rules” (p. 4). This is problematic since instructional practices that focus on memorization, rote procedures, and speed can result
in negative consequences for student learning and achievement (Boaler & Zoido, 2016). Instead, students need opportunities to represent, explore, and manipulate numbers in a flexible capacity using various strategies and approaches to challenge their thinking (Boaler & Zoido, 2016).

In a comprehensive review of 23 research articles on number sense instruction in schools, Sood and Mackey (2014) established three instructional approaches for teaching number sense to students: explicit instruction interventions, constructivist interventions, and a combination of explicit instruction and constructivist intervention. While researchers recommend that students experience interventions that are both procedural and constructivist in nature, Sood and Mackey (2014) found that only 13% of the studies relied on a combination of both methods. Moreover, only two studies examined the effects of number sense instruction on students at risk for acquiring mathematical proficiency. These findings implicate the need for additional research concerning the impact of number sense instruction on the development of mathematical proficiency in students (Sood & Mackey, 2014).

Boaler (1998) conducted concurrent case studies at two schools with similar demographics to compare the mathematical understanding of students predominantly subjected to a traditional or textbook approach to mathematics instruction versus students primarily exposed to a constructivist form of instruction. Boaler (1998) found the disparities in instructional approaches cultivated different kinds of knowledge in students. The results of the study revealed:
Students who followed a traditional approach developed a procedural knowledge that was of limited use to them in unfamiliar situations. Students who learned mathematics in an open, project-based environment developed a conceptual understanding that provided them with advantages in a range of assessments and situations. The project students had been “apprenticed” into a system of thinking and using mathematics that helped them in both school and nonschool settings. (Boaler, 1998, p. 41)

Boaler’s (1998) research established that traditional instructional approaches in mathematics were not successful in preparing students for the level of rigor required for solving real-world tasks. Furthermore, students who experienced traditional methods were no more prepared for assessments than students in progressive classrooms (Boaler, 1998). In fact, students who partook in student-centered instruction were better prepared for mathematics assessments and pragmatic situations and exhibited positive attitudes towards mathematics than students who were not exposed to student-centered instruction (Boaler, 1998).

In a study of 72 children ages seven to twelve, Gray and Tall (1994) analyzed students’ strategies for solving rudimentary arithmetic problems in a flexible manner. The researchers arranged the students into three categories—above average, average, and below average—based on their level of achievement in arithmetic. After interviewing participants over a two-month period, Gray and Tall (1994) recognized a noticeable difference between students identified as below level and above level and the strategies they used to solve simple addition and subtraction problems. Students in the above-
average group were inclined to calculate solutions using strategies such as “18 – 9 is 9 because 9 x 2 is 18” and “8 + 6 is 14 because two 7s are 14” (Gray & Tall, 1994, p. 129), while students in the below-average group were likely to solve problems such as 19 – 17 by “laboriously counting back 17 from 19” (Gray & Tall, 1994, p. 135). Gray and Tall (1994) coined the phrase *proceptual divide* to describe the differences in mathematical thinking between the two groups of students. Students in the high-performing group relied on *proceptual thinking*, the “meaningful use of known facts to arrive at solutions through derived facts” (Gray & Tall, 1994, p. 129), whereas low-performing students were dependent on procedural counting strategies, such as count all or count back. The high-achieving students used their sense of number to compute problems flexibly, while low-achieving students were unable to manipulate numbers and apply flexible strategies (Gray & Tall, 1994).

Right answers in mathematics do not indicate proficiency or understanding of the subject (Richardson, 2004). Students who are unable to express the relationships between numbers and operations or make connections between problems demonstrate limited number sense, even if they are able to procedurally solve computation problems and obtain the correct answers (Richardson, 2004). Although memorization of rules and procedures may appear successful for some, few are able to apply their knowledge to real-world situations (Parrish, 2014). As stated by Richardson (2004), “If children are going to be successful in the study of mathematics throughout their schooling, it is vital that the mathematics they learn be meaningful to them” (p. 53). Number sense is the foundation for meaning and understanding in mathematics.
Mental Computation

It is widely accepted that mental computation is a means of developing number sense in students (NCTM, 2000; NRC, 2001; Parrish, 2011, 2014; Reys, 1985; Reys, 1984). Reys (1985) defined mental computation as the ability to mentally “calculate exact numerical answers without the aid of any calculating or recording device” (p. 43). There are several benefits to the inclusion of mental computation in the mathematics classroom. Mental computation places emphasis on the relationships of numbers and operations, place value concepts, efficiency and flexibility in problem solving, estimation, and reasoning in mathematics (NCTM, 2000; NRC, 2001; Parrish, 2011, 2014; Reys, 1985; Reys, 1984). Reys (1984) identified seven instructional characteristics associated with mental computation: (a) “instruction must include different numbers (whole numbers, fractions, and decimals) and various operations with them”; (b) “instruction must be built on a framework the provides for systematic development of mental computation throughout the grades”; (c) “specific mental computation procedures must be carefully and meaningfully taught, so they are understood by all students”; (d) “procedures should be presented and developed in a manner that provides for individual differences”; (e) “instruction should encourage students to ‘think aloud’ and to participate actively in the sharing of procedures”; (f) “technology, such as hand-held calculators and computers, should be utilized to develop and practice mental computation skills”; and (g) “opportunities to do mental computation orally must be provided” (pp. 550-551). While mental computation in the early grades is emphasized in students’ learning of basic math facts with the four operations, students rarely have opportunities to apply mental
computation strategies to more difficult problems (Reys, 1984). Consequently, it is essential that mathematics programs incorporate instructional routines that purposefully aid students in developing computational fluency (Reys, 1984).

Computational Fluency

The authors of *Principles and Standards for School Mathematics* defined computational fluency. They explained:

[Computational fluency] refers to having efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate flexibility in the computational methods they choose, understand and can explain these methods, and produce accurate answers efficiently. The computational methods that a student uses should be based on mathematical ideas that the student understands well, including the structure of the base-ten number system, properties of multiplication and division, and number relationships. (NCTM, 2000, p. 152)

Students who demonstrate fluency in mathematics can solve problems efficiently, accurately, and flexibly (NCTM, 2000). Efficiency in mathematics refers to a student’s ability to solve problems easily using logic or sense making; students who are efficient in mathematics are able to execute appropriate strategies properly to find a solution (Russell, 2000). Accuracy denotes solving problems precisely; students who solve problems accurately work meticulously to ensure proper arithmetic and recording of their work and use reasoning skills to determine if their answers are logical (Russell, 2000). Fluent mathematicians also use knowledge of the base-ten number system, knowledge of
mathematical properties, and knowledge of number relationships to check for accurate results (NCTM, 2000; Russell, 2000). Flexibility involves selecting an appropriate method to solve a problem; students who demonstrate flexibility in mathematics are able to approach problems from multiple perspectives and use various strategies to solve their work (Russell, 2000). Furthermore, students who are flexible in mathematics are able to explain their strategies and justify their reasoning (NCTM, 2000).

Mental and Flexible Strategies

If children develop number concepts and logico-mathematical knowledge through their own ability to think, the goals in arithmetic must be that they invent their own procedures for solving problems and construct a network of numerical relationships (Kamii, 1996, p. 101).

In the early years of learning, students conceptually develop an understanding for whole number computation with addition and subtraction (NCTM, 2000). Even in the primary grades, instructional practices should promote strategies for manipulating and calculating numbers that support young learners in flexible thinking that produce computational fluency (NCTM, 2000). Thompson (1999) defined mental computation strategies as the “application of known or quickly calculated number facts in combination with specific properties of the number system to find the solution of a calculation whose answer is not known” (p. 2). For example, to find the sum of 5 + 6, a student might decompose the addend 6, renaming the expression as 5 + (5 + 1). Using the associative property, the problem can be rewritten as (5 + 5) + 1. In this case, the student applies his or her knowledge of 5 + 5, a doubles addition fact, to solve the problem. If 5 + 5 equals 10, then 5 + 6 equals 11 since it is only one more.
According to Madell (1985), “The early focus on memorization in the teaching of arithmetic thoroughly distorts in children’s minds the fact that mathematics is primarily reasoning. This damage is often difficult, if not impossible, to undo” (p. 22). Kamii and Dominick (1998) asserted that algorithms are harmful to the understanding of mathematical concepts as they eliminate students’ autonomy to construct their own meaning of number, and they do not reinforce place value, thus hindering students’ development of number sense. Parrish (2014) observed the harmful effects of algorithms on students’ understanding of subtraction during a classroom visit with second-grade students. Parrish (2014) described the following scenario:

I watched Melanie subtract 7 from 13. She had written the problem vertically on her paper and began solving it using the standard U.S. algorithm for subtraction. I asked Melanie to share her thinking about this problem. She said, “I couldn’t take seven from three so I borrowed ten. And I made the one a zero and three became a thirteen. And thirteen minus seven is six.” When asked why she chose to solve the problem this way, Melanie replied, “That’s just how you do it when the bottom number is bigger than the top.” (p. 3)

In a third-grade classroom, Parrish (2014) watched as students solved the problem 328 – 69 using the standard U.S. algorithm. Parrish (2014) reflected:

I was curious about their understanding of this process and posed, “When you crossed out the numbers in three hundred twenty-eight and wrote different numbers above it, did you still have an amount that is the same as in three hundred twenty-eight?” The students looked puzzled by this question and
collectively replied, “No, you have a different amount, because you have a two, an eleven, and an eighteen. That’s why you can minus the six and the nine.”

Their struggle to make the connection between decomposing or breaking apart 328 into 200 + 110 + 18 was an indication that they did not fully understand the method they had been taught. (pp. 3-4)

The mathematical reasoning of the students in these examples illustrates a lack of number sense as rules and procedures are replicated with no indication of conceptual understanding.

Kamii, Lewis, and Livingston (1993) claimed that student-invented strategies allow students to produce their own thinking, strengthen their understanding of place value concepts, and promote number sense. Researchers discerned that students who grapple with computation problems and develop their own sense-making strategies are likely to add and subtract from left to right when the problem is written in vertical form (Madell, 1985; Kamii et al., 1993). Madell (1985) reported two examples of strategies that seven- and eight-year-old students used to find the difference of 53 – 24. In the first example, a student subtracted the tens column first: 50 – 20 = 30. The student then subtracted: 30 – 4 = 26. In the final step, the student added 26 + 3 and concluded that 53 – 24 = 29. In the second example, a student explained that “50 minus 20 is 30. 3; 4. That’s 1 more to take away. 30 minus 1 is 29” (Madell, 1985, p. 21). Furthermore, Kamii and colleagues (1993) found that when students used invented strategies to solve multiplication problems, they also worked from left to right. In the problem 125 x 4, for example, a student decomposed the factor 125 into 100 + 20 + 5. The student then
proceeded to find the partial products: $4 \times 100 = 400$, $4 \times 20 = 80$, and $4 \times 5 = 20$. After adding the partial products together ($400 + 80 + 20$), the student determined that the product of $125 \times 4$ is equal to 500. Another student solved the same problem by decomposing the factor 125 into $100 + 25$. The student used the distributive property to find the product: $(4 \times 100) + (4 \times 25) = 400 + 100 = 500$.

While student-invented strategies encourage flexibility and sense making in mathematics, efficiency in calculating numbers is equally important as “some methods are more suited to some problems than others” (Threfall, 2002, p. 31). For example, Threlfall (2002) explained,

The adding of one number to another first by adding the tens and then the ones separately is more suited to $55 + 24$ than to $33 + 29$, whereas rounding a number before adding, and then compensating, is more suited to $33 + 29$ than to $55 + 24$.

A child who makes “good choices” of this kind is expected to be more effective in calculating. (p. 31)

Students who demonstrate computational fluency do not sift through a list of learned strategies, settling on an option for finding the solution. Instead, when students encounter new problems, they reflect on the connections between the numbers and select appropriate strategies, such as decomposing or rounding, based on their knowledge of number relationships (Threlfall, 2002).

As students invent their own strategies to solve problems, it is important that students verbally share their thinking with peers in a risk-free environment (Kamii et al., 1993; Parrish 2014). After grappling with a problem, it is appropriate for the teacher to
request that students share their solutions. It is important that all student answers be recorded—both correct and incorrect—to allow for meaningful discourse and to clarify misconceptions (Kamii et al., 1993; Parrish 2014). As the teacher facilitates the discussion, all students in the class are expected to engage in the conversation and contribute their ideas. As stated by Kamii and colleagues (1993),

The exchange of points of view is very important in a constructivist program, and the teacher is careful not to reinforce right answer or to correct wrong ones. If the teacher were to judge correctness of answers, the children would come to depend on him or her to know whether an answer is correct. If the teacher does not say that an answer is correct or incorrect and encourages the children to agree or disagree among themselves, the class will continue to think and to debate until agreement is reached. (p. 201)

Students who communicate their computation strategies and explanations with their peers benefit by (a) clarifying or refining their own thought processes, (b) examining the reasonableness of multiple strategies, (c) reflecting on mathematical relationships, (d) constructing a toolbox of efficient strategies, and (e) selecting appropriate methods to solve problems based on number relationships (Parrish, 2014).

Mathematical Discourse

Communication through classroom discourse communities has been part of mathematics reform efforts since the late 20th century (NCTM, 1989, 1991, 2000). Discourse, as defined by NCTM (1991), “refers to the ways of representing, thinking,
talking, and agreeing and disagreeing that teachers and students use to engage in . . .
tasks” (p. 4). According to NCTM (1991),

Students must talk, with one another as well as in response to the teacher . . .

When students make public conjectures and reason with others about
mathematics, ideas and knowledge are developed collaboratively, revealing
mathematics as constructed by human beings within an intellectual community.

(p. 34)

Effective discourse in mathematics necessitates that students construct arguments
to validate their own thinking in addition to interpreting, evaluating, and questioning the
reasoning of peers (Bennett, 2014; CCSSI, 2018a). NCTM (1991) identified seven roles
of the student in mathematical discourse: (a) “listen to, respond to, and question the
teacher and one another”; (b) “use a variety of tools to reason, make connections, solve
problems, and communicate”; (c) “initiate problems and questions”; (d) “make
conjectures and present solutions”; (e) “explore examples and counterexamples to
investigate a conjecture”; (f) “try to convince themselves and one another of the validity
of particular representations, solutions, conjectures, and answers”; and (g) “rely on
mathematical evidence and argument to determine validity” (p. 45). Stein (2007)
acknowledged that both discourse and conceptual understanding are equally important in
learning mathematics. As Lampert (1990) conveyed,

Mathematical discourse is about figuring out what is true, once the members of
the discourse community agree on their definitions and assumptions. These
definitions and assumptions are not given, but are negotiated in the process of determining what is true. (p. 42)

Consequently, instructors must provide opportunities for students to discuss, question, and investigate problems and strategies in a comfortable environment (Parrish, 2011, 2014).

Classroom Culture and Social Relationships

For students who lack previous experiences with discourse, the practice of communicating mathematical thinking can present an uncomfortable situation filled with uncertainty (Hufferd-Ackles, Fuson, & Sherin, 2004). Thus, the culture of the classroom sets a precedence for student risk-taking in discourse communities. Hufferd-Ackles and colleagues (2004) observed a group of third-grade students new to mathematical discourse for a period of one school year and noted:

Initially, standing in front of one’s peers to communicate mathematical ideas was a daunting task for many students, especially shy students. Many chose to sit back down in their seats after writing work on the board rather than to accept the challenge of staying in front of the class and talking. However, as the math-talk learning community developed, students’ attempts at explaining were scaffolded by supportive classroom colleagues. This support allowed the development of explaining to progress. As students learned to explain their own mathematical thinking more fully and fluidly, they made significant contributions that could then be questioned or built on by other students and assessed by the teacher. (p. 97)
NCTM (2000) ascertained, “Teachers establish and nurture an environment conducive to learning mathematics through the decisions they make, the conversations they orchestrate, and the physical setting they create” (p. 18). Student relationships are central to this environment (Meltzoff, 1994). Although classrooms are filled with social experiences for students, Meltzoff (1994) cautioned against the assumption that this constitutes a community of learners.

Bennett (2014) noted that the establishment of norms early in the school year sets clear expectations regarding classroom culture and participation. While the development of classroom norms is commonplace in most classrooms, Yackel and Cobb (1996) found that the social norms developed for general classroom practice were not sufficient for the discourse needs of mathematics. As a result, they (1996) coined the term *sociomathematical norms* to differentiate between classroom social norms and norms appropriate for mathematical discourse. Yackel and Cobb (1996) noted a discernable difference between the two types of norms, defining social norms as “the understanding that students are expected to explain their solutions and their ways of thinking” (p. 461) and sociomathematical norms as “understanding of what counts as an acceptable mathematical explanation” (p. 461). The classroom teacher and students, in tandem, establish sociomathematical norms, defining quality discourse contributions and modifying the norms as interactions evolve. For example, members of the classroom community may agree that contributions to the discussion should be authentic and that contributors should not depend on cues from peers to determine correctness. These norms require students to generate their own conceptual reasoning for solving problems,
rather than relying on regurgitation of procedural knowledge and the influences of peers (Yackel & Cobb, 1996).

Lloyd, Kolodziej, and Brashears (2016) proposed the Facilitate—Listen—Engage (FLE) model for discourse communities. In the first phase of this model, the teacher, or facilitator, intentionally designs instruction to engage students in rich discourse. In the listen phase, both facilitator and students communicate information and ideas. In the final phase, the engage phase, the facilitator presents opportunities for students to connect with peers and engage in conversation that inspires classroom community (Lloyd et al., 2016). The FLE framework, which is inclusive of all students, conveys that each member of the class is vital to the learning process (Lloyd et al., 2016).

Student talk and interaction is a principal component for initiating classroom discourse and increasing students’ knowledge, understanding, and learning in mathematics (Franke et al., 2009). Franke and colleagues (2009) identified two interdependent benefits of student interaction in mathematics. First, teachers who listen to student dialogue are able to monitor students’ mathematical thinking and design instructional practices to meet the needs of the students. When students interact in mathematical conversations, they are able to gauge the strategies of their peers and provide feedback that assists in building more in-depth mathematical understanding. Second, the act of talking and interacting assists students in developing understanding by describing, explaining, and justifying their thinking to their peers. Talking allows students to reorganize and clarify their thoughts while acquiring new strategies and knowledge that produce understanding (Franke et al., 2009).
For some teachers, the nature of discourse practices presents an overwhelming experience (Hufferd-Ackles et al., 2004). To support teachers in developing a discourse community, Hufferd-Ackles et al. (2004) created a guiding framework for a math-talk learning community, or “a classroom community in which the teacher and students use discourse to support the mathematical learning of all participants” (p. 82). In a math-talk learning community, the main goal is to “understand and extend one’s own thinking as well as the thinking of others in the classroom” (Hufferd-Ackles et al., 2004, p. 82). Hufferd-Ackles et al. (2004) identified four distinct, but interrelated components necessary for building a classroom discourse community: “questioning”, “explaining math thinking”, “source of mathematical ideas”, and “responsibility for learning” (p. 87).

Within each component are four developmental levels, on a scale of 0 to 3, that demonstrate growth in cultivating a math-talk learning community. At level 0, the teacher embraces a traditional, teacher-centered mathematics classroom, while at level 3, the teacher facilitates student discourse and supports students as they assume leadership in the conversation. Movement through the levels is dependent on growth in mathematical practices over time. Stein (2007) cautioned, “[The] framework serves as a good indicator for assessing the discourse level of the whole class, [but] it does not assess individual students” (p. 288). Consequently, teachers must monitor the progress of individual students’ participation and support learners struggling with the process (Stein, 2007). The framework design benefits teachers who aspire to nurture communication in mathematics by delineating teacher and student roles in a discourse community.
The Teacher’s Role

In 1991, NCTM identified seven distinct roles of the teacher as facilitator in mathematical discourse: (a) “posing questions and tasks that elicit, engage, and challenge each student’s thinking”; (b) “listening carefully to students’ ideas”; (c) “asking students to clarify and justify their ideas orally and in writing”; (d) “deciding what to pursue in depth from among the ideas that students bring up during a discussion”; (e) “deciding when and how to attach mathematical notation and language to students' ideas”; (f) “deciding when to provide information, when to clarify an issue, when to model, when to lead, and when to let a student struggle with difficulty”; and (g) “monitoring students’ participation in discussions and deciding when and how to encourage each student to participate” (p. 35). The role of the teacher in discourse is often misinterpreted, for some assume that students will miraculously meet the intended objectives if students are engaged in meaningful tasks and the teacher has little involvement in the process (Chazen & Ball, 1995). This misconception undermines the importance of the teacher in facilitating worthwhile discussions.

Stein (2007) used the terms cognitive discourse and motivational discourse to describe teachers’ roles in discourse. Stein (2007) defined cognitive discourse as “what the teacher says to promote conceptual understanding of the mathematics itself” (p. 286), whereas motivational discourse “refers not only to praise offered to students but also to supportive and non-supportive statements teachers make that encourage or discourage participation in mathematics classroom discussions” (p. 287). Turner, Meyer, Midgley, and Patrick (2003) studied the motivational discourse of two 6th-grade teachers and its
impact on mathematics students. The researchers found that, in both classrooms, supportive discourse that highlighted conceptual understanding had a positive effect on students’ self-regulation and coping mechanisms. An analysis of supportive motivational discourse between the two teachers revealed interesting results—in one classroom, the teacher cultivated a classroom community of autonomous and collaborative learners, while the teacher in the other classroom displayed less supportive motivational discourse. The results indicated that students in the classroom with less supportive motivational discourse were more likely to exhibit behaviors of avoidance and negativity (Turner et al., 2003).

Mathematical language. The presence of formal language in mathematics is critical to precisely communicating thinking (NCTM, 2000). As identified by NCTM (1991), one of the roles of the teacher as discourse facilitator is to connect students’ ideas strategically with precise mathematical language. In a study conducted over a three-month observational period at a school located in a low-income, predominantly African-American neighborhood, Cooke and Buchholz (2005) focused on the communication strategies of a kindergarten teacher that stimulated the use of formal math language with young learners. The teacher’s instructional practices consistently provided opportunities for students to engage in dialogue. During small group instruction, the teacher listened to students’ conversations and asked questions to facilitate their thinking. The teacher promoted mathematical connections by allowing for exploration, accessing prior knowledge, asking questions, incorporating appropriate vocabulary, and facilitating discourse. Through conversations that related mathematical concepts and terminology to
real-world tasks, the teacher was able to generate a community of learners who were eager to explore and participate in verbal communication. The findings of the study revealed that purposeful discourse, skillfully facilitated by the teacher, can stimulate students’ reasoning and cultivate formal math language in young students (Cooke & Buchholz, 2005).

Questioning. The role of teacher questioning elicits the mathematical responses students convey during mathematical discourse. Hiebert and Wearne (1993) stressed:

The most compelling theoretical argument in favor of higher-order questions is that if students are challenged to explain the reasons for their responses or define their positions, they will engage in deeper, reflective, integrative thought than if they are asked to recall facts or rules. (p. 397)

Franke and colleagues (2009) conducted a study that involved the ways that teachers questioned students in order to extend their mathematical thinking. The study consisted of two second-grade teachers and one third-grade classroom teacher who taught similar concepts and skills and demonstrated similar classroom structures. The researchers videotaped and audio recorded conversations and analyzed the relationship between teacher instructional practices and student participation. Specifically, Franke and colleagues (2009) examined the types of questions posed by the teachers to investigate students’ ideas and thinking. The study consisted of two different parts: teacher questioning practices and questioning and student explanations.

In the first part of the study, the results showed teachers directed students to share their thinking in whole-group situations 98% of the time. Teachers inquired about
students’ explanations to seek understanding 76% of the time. Teachers asked questions concerning accurate responses 67% of the time; and 82% of the time, teachers prompted students to explain incomplete or imprecise responses (Franke et al., 2009). These results clearly indicate that teachers expected students to express their reasoning in mathematics.

In the second part of the study, the researchers found that when teachers did not ask students to elaborate on their explanations, only 32% of the students provided explanations independently, while 72% of the students provided elaboration if the teacher prompted follow-up questions. Additionally, the types of follow-up questions asked by the teachers (i.e., general questions, specific questions, leading questions) impacted the amount of elaboration provided in the students’ responses (Franke et al., 2009). The findings of the study reveal that teacher questioning is critical to student thinking and reasoning in mathematics (Franke et al., 2009).

Autonomous learners. As the instructional leader in mathematics, another role of the teacher is to develop autonomous and self-regulated learners, defined by Pape, Bell, and Yetkin (2003) as learners who “are active participants in their own learning, are able to select from a repertoire of strategies and to monitor their progress in using these strategies toward a goal” (p. 179). According to Lampert (1990), all mathematics students “should be making conjectures, abstracting mathematical properties, explaining their reasoning, validating their assertions, and discussing and questioning their own thinking and the thinking of others” (pp. 32-33).

Zimmerman (2002) identified three phases of self-regulated learning: (1) forethought, (2) performance, and (3) self-reflection. Zimmerman’s (2002) first phase,
forethought, requires learners to analyze tasks and set goals for themselves. The second phase, performance or volition control, involves students monitoring and controlling their knowledge of content; in this stage, students use metacognition to evaluate and determine their learning needs. The third phase, self-reflection, necessitates that students evaluate their progress and modify their learning accordingly (Zimmerman, 2002).

In a study conducted by Pape and colleagues (2003), a seventh-grade mathematics teacher and a university faculty member worked collaboratively to cultivate the mathematical thinking and self-regulated learning of middle-school students. During Pape et al.’s (2003) yearlong study, the instructors used multiple representations and meaningful mathematical tasks to engage students in problem solving. The tasks encouraged students to generate complex representations that promoted rich mathematical thinking. As students completed the tasks, the teacher questioned students to elicit reasoning and provided support by connecting mathematical ideas. Pape et al. (2003) discovered, “While students provided evidence that their knowledge and awareness of strategies had increased, their volitional control was too limited to sustain their use of strategies” (p. 196). However, Pape et al. (2003) noted that students succeeded in effectively communicating their strategies, reasoning, and understanding of mathematics.

Tasks, knowledge, and management. Traditionally, the teacher’s role in the mathematics classroom has been one of telling, confirming right and wrong answers without opportunities for students to engage in conversation about problems and solutions (Lampert, 1990). In a case study focused on teacher-student interactions in a
fifth-grade classroom, Lampert (1990) redefined the typical classroom structure for knowing mathematics:

I gave them problems to do, but I did not explain how to get the answers, and the questions I expected them to answer went beyond simply determining whether they could get the solutions. I also expected them to answer questions about mathematical assumptions and the legitimacy of their strategies. Answers to problems were given by students, but I did not interpret them to be the primary indication of whether they knew mathematics. (p. 38)

In an effort to promote mathematical discourse among students, Lampert (1990) purposefully selected problems that could not be solved with a known algorithm. Accordingly, students had to devise a plan for solving the problem in addition to finding the solution. As students shared their answers, Lampert recorded each solution on the board (right or wrong) with a question mark and the student’s name. At this point, students were able to defend their solutions by explaining their reasoning, or they could respectfully disagree or question a peer’s answer. During this process, Lampert (1990) described the teacher’s role in the discourse community as follows:

The role I took in classroom discourse . . . was to follow and engage in mathematical arguments with students; this meant that I needed to know more than the answer or the rule for how to find it, and I needed to do something other than explain to them why the rules worked. I needed to know how to prove it to them, in the mathematical sense, and I needed to be able to evaluate their proofs of their own mathematical assertions. In the course of classroom discussions, I
also initiated my students into the use of mathematical tools and conventions. Information about tools and conventions was integrated with teaching the class about the process of doing mathematics. (p. 41)

Over the course of the study, Lampert noted a few challenges that arose in the discourse community. First, students not confident in their abilities were inclined to agree with peers who they perceived as smart in mathematics, often making remarks such as “because that’s what Tommy said, and he’s usually right” (Lampert, 1990, p. 56). Second, Lampert observed that students deemed successful in memorizing and following rules were often disengaged, and at times disruptive, in the conversation, feeling no obligation to listen or debate the reasoning of peers. Third, Lampert (1990) found that students uncomfortable or inexperienced with sharing mathematical thinking tended to explain their reasoning as “I just know”, or “I just thought it”, or “I don’t know how I figured it out” (p. 56). Last, Lampert (1990) revealed that some students attempted to intimidate peers who challenged their solutions, despite this behavior defied the community norms. Nevertheless, Lampert (1990) found that overall, students’ attitudes regarding what it means to know and do mathematics shifted, and they exhibited a different outlook on mathematics learning as compared to students in traditional settings.

Meta-Analysis of Mathematical Discourse

In a meta-analytic study of 108 journal articles on discourse in mathematics education, Ryve (2011) found that 78% of the articles reviewed discourse data in terms of talk, while only 9% of the research focused on written text. Of the articles that concentrated on talk as data, 89% used ordinary text transcripts, whereas only 11% used
coding in the transcripts. This is significant since 85 of the 108 articles used discourse in terms of social interaction or speech. Ryve (2011) suggested:

By combining these two findings, one could claim that it is likely that many articles would benefit from using more detailed transcripts to be able to capture interactional aspects in mathematics classrooms. Further, whether or not more detailed transcripts are needed, one may argue that the quality of the article would increase if explicit discussions of the rationale for choosing ways of transcribing verbal talk were included. (p. 185)

Upon analysis of the types of mathematical discourse that occurred, Ryve (2011) discovered that only 48% of the studies emphasized mathematical constructs, while 52% concentrated on generic discourse in education. The results of this meta-analysis indicate a need for future research on discourse to include a thorough review of transcripts in an effort to understand the social interactions in the mathematics classroom. Furthermore, additional research on the conceptualization of mathematics as a discourse is necessary.

Empirical Literature Search Strategy

To investigate the role of math talks and classroom number talks in developing number sense, mental computation, and discourse in elementary mathematics students, an extensive review of the existing literature was conducted to evaluate the scope of research most relevant to the topic. Multiple electronic databases were accessed to locate prominent articles associated with each theme. These databases included EBSCO Education Full Text, Education ProQuest, ERIC, JSTOR, and ProQuest Dissertation and Theses. To find the literature most pertinent to this study, a systematic search was
performed using keywords identified in related research studies. Search terms included: *mathematics, number sense, number talks, math talks, conceptual understanding, discourse, classroom culture, classroom environment, learning communities, mental computation, computational fluency, teacher questioning, student reasoning, and critical thinking*. In addition, web searches were conducted with the terms to access documents available on the Internet. Article titles and abstracts were reviewed to determine their relevance to the research topic. Consultation with professors and experts in the field of mathematics assisted in narrowing the search results and acquiring seminal works. The search for quality literature involved a thorough review of the references included in the research articles gathered from the databases. Studies included as a reference in previous research articles were obtained through additional searches and reviewed for applicable information. Subsequently, a comprehensive inventory of sources critical to the study was compiled and distributed to professionals who reviewed the references and offered further recommendations. Finally, research selected for inclusion in the literature review had to meet specific search parameters such as peer-reviewed journals, research studies, and scholarly articles and books. Editorial articles that expressed personal views or nonresearched based strategies or suggestions were not included in the literature review. Last, the literature had to be available in English and published within the past 30 years.

**Classroom Number Talks: Empirical Research**

Students who strictly rely on memorized procedures to solve problems in mathematics often demonstrate mediocre number sense. O’Nan (2003), concerned for the lack of number sense exhibited by 22 fourth-grade students, implemented classroom
number talks with concentration on two-digit addition for approximately 10 minutes per day for a six-week period to develop mental computation strategies and to cultivate number sense in students.

Prior to implementation, a two-fold pretest interview was administered to each participant. Initially, O’Nan (2003) displayed a two-digit addition problem and prompted each participant to share multiple strategies for mentally calculating the problem. Next, O’Nan provided each participant with a set of two-digit addition flashcards. During this portion of the interview, students verbally reported their solution to each problem, and O’Nan recorded the number of accurate answers provided by each participant in a two-minute timeframe.

Following implementation of the number talks treatment, a posttest, identical to the pretest, was administered to each participant, and O’Nan (2003) conducted a paired $t$-test using the results of the pretest and posttest data. A comparison between the number of strategies participants were able to produce between the pretest ($M = 1.45$) and posttest ($M = 3$) revealed a statistically significant difference. O’Nan (2003) reported, “Daily discussion of various strategies more than doubled the mean number of strategies the children could produce” (p. 34). Likewise, a comparison between the number of two-digit addition problems that students were able to accurately answer in a two-minute period between the pretest ($M = 9.17$) and posttest ($M = 10.59$) proved statistically significant.

Additionally, O’Nan (2003) conducted two correlation tests to determine if a correlation exists between the number of strategies generated by participants and the
number of problems solved accurately in two minutes by participants. The results yielded no significant gains. The findings of O’Nan’s (2003) study indicate that exposure to daily number talks influenced flexible thinking in students when applying mental strategies to solve two-digit addition computation problems.

Celski (2009) qualitatively researched the effects on first-grade students’ number sense and participation in mathematics instruction as a result of daily number talks implementation. Participants completed a pre- and posttest consisting of 25 computation problems. After each test administration, Celski scored the assessments and calculated a percentage to represent the participants’ computation fluency. Celski placed assessment results of 36 participants in a table for comparison and determined that student improvement was evident if the participant’s percentage score increased between administration of the pretest and posttest. Results of the study showed that 25 students demonstrated growth between the pre- and postassessment. Furthermore, Celski (2009) tracked the participation of 18 students during mathematics instruction. Results indicated that 16 of the students exhibited increased participation in mathematics instruction due to exposure of classroom number talks.

Johnson and Partlo (2014) conducted a mixed-methods research design to investigate the mental computation skills of fourth-grade students in two urban elementary schools after participation in classroom number talks. Prior to the number talks intervention, the researchers administered a four-statement questionnaire using a Likert scale to gather data on the participants’ attitudes and beliefs about solving mental computation problems. Additionally, Johnson and Partlo (2014) administered a pretest
that included a total of six double-digit addition and double-digit subtraction problems. Participants were instructed to use mental strategies to solve the computation problems, listing only a solution and explanation of the strategy on the recording sheet.

Over the course of a two-month period, students participated in 10 classroom number talks using predetermined number strings designed by the researchers to emphasize five addition and five subtraction strategies (Johnson & Partlo, 2014). Following the implementation of each number talk, the researchers recorded anecdotal notes concerning misconceptions, patterns, and observations of the participants. Furthermore, the researchers conducted 12 student interviews to acquire a broader understanding of the participants’ perspectives on the effectiveness of number talks. At the conclusion of study, the researchers administered a postquestionnaire and posttest and analyzed variation in participant responses after receipt of the intervention.

Johnson and Partlo (2014) found that regular participation in classroom number talks had a positive impact on the mental computation skills of fourth-grade students. An analysis of participant responses on the pre- and postquestionnaire and the pre- and posttest revealed that, following the intervention period, students were able to articulate the strategies they used to solve problems instead of relying on drawings, examples, and words. A comparison of participant scores between the double-digit addition and subtraction problems showed a noticeable difference in student performance. Scores on both the pre- and posttest indicated that participants were more comfortable with mentally solving addition problems than mentally solving subtraction problems. The
results of the assessments were consistent with observations made by the researchers during classroom number talks. Johnson and Partlo (2014) reported,

> It was not uncommon for students to come up with three different answers during a subtraction number talk. They often tried methods that were not appropriate for the problem and had a difficult time transferring recently learned strategies to new problems. (p. 25)

Although subtraction was a challenge for students, Johnson and Partlo (2014) noted improvement in participant responses on subtraction problems as number talk implementation continued. Johnson and Partlo (2014) observed, “Students seemed to be more proficient during the final subtraction number talks, frequently using strategies such as Counting Back and Adding Up” (p. 25). Furthermore, participants demonstrated growth in mentally solving subtraction problems between the pre- and posttest, with an average increase of 36 percent. Moreover, all 12 participants who were interviewed concerning their experiences with number talks reported an increase in proficiently solving problems using mental strategies (Johnson & Partlo, 2014).

Clark (2015) studied the effects of number talks on fostering student autonomy among a group of 27 third-grade students in a middle-class, suburban community. Over a 10-week period, the researcher collected data by means of student reflections, surveys, video recording, transcriptions, and teaching logs. Following each number talk, Clark (2015) administered a survey to each participant to assess personal engagement, learning, and knowledge. Clark then reviewed the data from the surveys to determine the engagement levels of students during number talks and to identify strengths and
weaknesses of students’ learning and knowledge. At the conclusion of the research period, Clark (2015) coded and analyzed the data to identify patterns and trends among participant responses. Additionally, the researcher examined, coded, and analyzed video recordings of the classroom number talks to evaluate patterns in teacher-student interactions and student-student interactions and its impact on student autonomy and engagement. The final data piece was a teaching log that included records on lesson preparations, observations, questions, and reflections. Clark (2015) compared data among the teaching log, videos, and participant surveys to distinguish differences between teacher perception and student perception.

Careful analysis of the data revealed four main themes: (a) “focused teacher preparation is key to the inclusion of autonomy-supportive instructional methods”; (b) “an open-ended questioning style is a powerful tool for increasing student engagement, autonomy, and participation”; (c) “student engagement and autonomy-supportive teacher behaviors increased over time”; and (d) “a semi-random method of selecting students to respond lead [sic] to greater and more even participation” (Clark, 2015, p. 26). Clark (2015) found that cultivating student autonomy was difficult since classroom routines and structures had yet to be firmly established. In particular, Clark recognized the need to define classroom norms for respectful discourse. Clark (2015) conveyed, “Two of the core changes that . . . supported student autonomy were creating more focused lessons, and asking more open-ended questions while also pushing students to explain their ideas” (p. 27). Additionally, students need time to explore, reason, and explain mathematical thinking (Clark, 2015).
Clark (2015) also found that open-ended questioning presented opportunities for students to become autonomous learners. Clark (2015) reported, “Asking open-ended questions is key to supporting student autonomy because students are offered opportunities to share and develop their thinking, and also begin to take more responsibility for explaining the mathematical concepts and determining the correct answer” (p. 30). Clarke (2015) concluded that the teacher must use autonomy-supportive language to develop autonomous learners.

Another finding of the study revealed that the more difficult the number talk, the more students were engaged in the lesson (Clark, 2015). Students’ survey responses on a number talk perceived difficult described the lesson as where they “had the most fun, worked the hardest, learned the most, and found the number talk most interesting” (Clark, 2015, p. 38). Finally, Clark (2015) reported, “As the study progressed, the percentage of responses by the most talkative students fell, while the percentage of responses by less talkative students rose” (p. 40). Thus, participation increased when semi-randomly selecting students to share their responses. Clark (2015) attributed the shift in participation to increased student self-efficacy as expectations for the learning community became more clearly defined.

Ruter (2015) used a quasi-experimental research design to investigate the effects of number talks on students’ number sense and critical thinking skills in mathematics. The study included 47 second-grade students from upper middle-class families at two suburban elementary schools. The experimental group consisted of 24 students, and the control group involved 23 students. Prior to starting the intervention, Ruter administered
a pretest to both groups of students that included two complex, critical-thinking problems, scored via a rubric created by Ruter. Following the four-week intervention period, a posttest was administered to both groups of students that consisted of two similar, but different problems.

Students in the experimental group received the number talks intervention two-to-three times per week, while students in the control group did not receive the treatment. At the conclusion of the study, Ruter (2015) used a paired t-test to compare data between the pre- and posttest results for both the experimental group and the control group to determine if student achievement of number sense and critical thinking increased because of the number talks intervention. An analysis of the data revealed no statistical difference between the experimental and control groups (Ruter, 2015).

Washington (2015) implemented a mixed-methods research design to determine if a correlation exists between the frequency of teacher-reported number talks implementation and student achievement on the 2014 State of Texas Assessment of Academic Readiness (STAAR) mathematics test for students in grades three through eight. Furthermore, Washington (2015) qualitatively examined teachers and administrators’ attitudes and beliefs about number talks. To determine the frequency that teachers reported implementing number talks in a one-week period and to gather information on teachers and administrators’ perceptions regarding the number talks strategy, teachers and administrators of grades kindergarten through eight completed a self-report questionnaire. Based on their responses to the self-report questionnaire, Washington (2015) categorized teachers into implementation frequency groups.
Teachers who reported number talks implementation zero-to-two times per week were identified as low-frequency, and teachers who reported implementation three-to-five times per week were labeled high-frequency.

Washington (2015) matched 37 classroom teachers’ number talks implementation frequency reports with students’ scale scores on the 2014 STAAR mathematics test in grades three through eight. Next, Washington (2015) conducted a one-way analysis of variance (ANOVA) for each grade level, three through eight, to establish if there was a statistical difference between the low-frequency and high-frequency groups, and to determine if a correlation exists between the frequency of number talks implementation and student achievement on the STAAR mathematics test. Washington (2015) reported a statistical difference in grade six:

The difference between means in the two groups was significant, $F (1,111) = 4.878$, $p = 0.29$, indicating that the null hypothesis is rejected, and it is concluded that the frequency of teacher usage had an impact on mathematics student achievement as measured by STAAR. Pearson $r$ analysis was also conducted to determine if there was a significant relationship between the two variables. The correlation was significant, $r (113) = .205$, $p = .029$, indicating a positive relationship exists between frequency of use (in days per week) and STAAR scale scores. (p. 116)

However, Washington (2015) found a negative correlation between frequency of number talks implementation and student achievement scores. Washington (2015) stated, “The teachers in the higher frequency group had lower scale scores than the teachers in the
lower frequency group” (p. 113). Moreover, Pearson’s $r (p > .05)$ indicated no relationship between the frequency of number talks implementation and student achievement ($p > .05$). Washington (2015) noted that due to the small sample size of the study, the results were inconclusive.

Washington (2015) also investigated teachers and administrators’ beliefs and attitudes regarding the number talks strategy. The results indicated that the majority of participants believed that the number talks strategy positively affects student achievement and aids students in solving mental computation problems and developing number sense. However, 30% of participants reported doubts concerning student success with number talks. When asked to describe their experiences with the number talks strategy, the data showed:

Some teachers reported the strategy helped students become faster with fact fluency, encouraged them to be more actively engaged, and promoted a wider variety of methods to solve problems. Other teachers reported the Number Talks strategy improved mental math and developed better reasoning. However, several teachers believed more training for teachers was needed, and follow-up workshops would better support teachers and implementation of the strategy. (Washington, 2015, p. 151)

Although results of the quantitative data were not conclusive, the qualitative data suggested that most teachers and administrators have a positive attitude toward the number talks strategy and believe it positively influences students’ understanding of mathematical concepts (Washington, 2015).
Okamoto (2015) conducted a mixed-methods design study to explore the implementation process and impact of number talks on sixth-grade students at a diverse urban middle school. Quantitatively, Okamoto (2015) measured the effects of number talks on students’ number sense development using a pre- and posttest design. Qualitatively, Okamoto (2015) studied the implementation of number talks to understand how they support students in strengthening number sense.

Over a six-week period, Okamoto (2015) facilitated a total of 15 classroom number talks that emphasized multiplication of whole numbers to a group of sixth-grade students. The study included 22 sixth-grade students who participated in a 15-minute number talk approximately two to three times per week. Prior to implementing the number talks intervention, Okamoto (2015) administered a 17-question pretest to students that consisted of three subsections: (a) algorithms, (b) equivalent expressions, and (c) mental math computation. The pretest measured (a) students’ proficiency in solving computation problems using the traditional subtraction, multiplication, and division algorithms, (b) students’ level of understanding with place value, the distributive property, and the doubling and halving strategy, and (c) students’ application of number concepts to solve multiplication problems using mental strategies.

Okamoto (2015) designed two number talk structures for implementation with students. The first structure consisted of number of the day problems where students were asked to share equivalent expressions for a number. For example, if the number of the day was 15, a student might share that an equivalent expression for 15 is $5 + 5 + 5$, $20 - 5$, or $3 \times 5$. The other number talk structure entailed mental computation, which
consisted of related, whole-number multiplication problems that students solved using mental math strategies. During each number talk, Okamoto (2015), as the facilitating teacher, recorded the strategies shared by students and produced visual representations where appropriate for understanding. The researcher recorded each number talk session, maintained a reflective journal, and collected student work samples to qualitatively analyze the data. Following the six-week implementation period, Okamoto (2015) administered a posttest to students. The posttest contained problems that were either the same or very similar to those included on the pretest.

To quantitatively analyze the data, Okamoto (2015) conducted one-tailed, matched pair t-tests. The results were statistically significant for the overall mean score on the assessment ($t = 2.48, p = 0.01, SD = 4.2$). Furthermore, the mean score for the equivalent expressions section of the assessment were statistically significant ($t = 4.28, p = 0.00, SD = 2.3$). Qualitatively, Okamoto (2015), identified three teacher practices that were important to the effective implementation of number talks: (a) reviewing student work samples to formatively assess student progress, (b) using visual models and representations to develop students’ understanding of the mathematics, and (c) selecting one mathematical idea or focus for each number talk session.

Can and Durmaz (2016) quantitatively studied the effects of number talks on number sense understanding over a three-week period. The participants of the study included 31 third-grade preservice primary teachers. The researchers used a single group pre- and posttest design to determine if the average scores of participants on a number sense test were statistically significant as a result of exposure to number talks. To score
the assessment, correct answers were assigned one point, and incorrect answers were assigned 0 points. The t-test results of average scores indicated no statistical difference in preservice teachers’ scores between pre- and posttest administrations as a result of number talks implementation ($t = 0.56, p > .05, M = 10.54, SD = 2.93$).

Additionally, Can and Durmaz (2016) investigated if a statistical difference existed in the participants’ usage of number sense between the pre- and posttest on the number sense test because of the intervention. Scores on the assessment were dependent on the level of number sense understanding displayed by the participants in their solutions and explanations. The t-test results of the participants’ number sense point averages indicated a significant increase in preservice teachers’ reliance on number sense to solve problems following the implementation of number talks ($t = 5.34, p = .000, M = 38.48, SD = 11.60$).

Turner (2017) used a mixed-methods design to examine the effects of number talks on student achievement. Study participants included one teacher and five second-grade students who qualified for the English to Speakers of Other Languages (ESOL) program. The researcher collected data through participant surveys, classroom observations, instructor conferences, student assessments, and teacher artifacts. Prior to starting the intervention, Turner (2017) administered a pretest comprised of five computational problems. Students were instructed to solve each problem using a preferred strategy.

Over a four-week period, students in the focal group participated in a number talk routine for 10 to 15 minutes at least three times per week. The focus of each number talk
was addition and subtraction, since this was a core standard for second-grade students. The researcher observed each number talk session and took anecdotal notes to record the students’ responses, engagement, and participation. At the end of each week, Turner (2017) met with the teacher participant to debrief and provide feedback on the process.

An analysis of the qualitative data revealed that the teacher participant implemented number talks effectively in a small-group setting, engagement of the student participants contributed to increased understanding of the content, and inclusion of visual representations and multiple strategies were critical in supporting students’ confidence with problem solving. Quantitatively, Turner (2017) used a pre- and posttest design to determine if the implementation of number talks increased the achievement level of the student participants. Results of a paired t-test indicated a statistically significant difference in the overall mean score between the pre- and posttest ($t = 6.53, p = 0.00, M = 76, SD = 16.73$). Following the study period, the researcher asked all participants to complete a Likert-scale questionnaire using a scoring range of one to five, where one was the lowest rating and five was the highest rating, to evaluate their experience with number talks. Sixty percent of the students were very satisfied with the practice, and 40% reported they were extremely satisfied with the practice. The teacher participant indicated that she enjoyed the experience but would appreciate additional professional learning support with teaching mathematics.

Tables 2 through 4 offer summaries of number talks research in this literature review. Table 2 provides quantitative empirical research on number talks. Table 3 displays qualitative empirical research. Table 4 presents mixed methods research.
Table 2

**Quantitative Empirical Research on Number Talks**

<table>
<thead>
<tr>
<th>Researcher(s)</th>
<th>Setting/Participants</th>
<th>Methods</th>
<th>Treatment</th>
<th>Study Findings</th>
</tr>
</thead>
</table>
| O’Nan (2003)  | • Large, Suburban Elementary School  
• East Tennessee  
• 22 fourth graders  | Pretest-posttest design | Exposure to daily number talks for a six-week period | • Statistically significant: number of strategies produced between the pre- and posttest  
• Statistically significant: number of two-digit addition problems solved correctly between pre- and posttest |
| Ruter (2015)  | • Two suburban public elementary schools  
• near Denver, CO  
• 47 second graders  | Quasi-experimental design | Exposure to number talks 2-3 times per week over a 4-week period | • No statistical difference between the experimental group and control group |
| Can & Durmaz (2016) | • State university  
• 31 third-grade preservice teachers  | Pretest-posttest design | Exposure to number talks over a 3-week period | • No statistical difference in pre- and posttest scores  
• Statistically significant: number sense point average scores between pre- and posttest |

Table 3

**Qualitative Empirical Research on Number Talks**

<table>
<thead>
<tr>
<th>Researcher</th>
<th>Setting/Participants</th>
<th>Methods</th>
<th>Treatment</th>
<th>Study Findings</th>
</tr>
</thead>
</table>
| Celski (2009) | • Elementary school  
• suburban Eastern Washington  
• 36 first graders  | Comparison of pre- and posttest data  
• Teacher survey and observations  | Exposure to daily number talks | • Increased number sense  
• Increased student participation  
• Increased score from pre-to posttest  
• No increase in fluency  
• Teacher preparation is necessary for effective implementation of student-autonomous instructional methods.  
• Open-ended questions increase student engagement.  
• Student engagement and supportive teacher behaviors increased over time. |
| Clark (2015)  | • Public elementary school  
• small, suburban community  
• 27 third graders  | Student reflections and surveys  
• Video recordings and transcriptions  
• Teaching log  | Exposure to number talks 2-3 times per week for a 10-week period to support student autonomy |   |
<table>
<thead>
<tr>
<th>Researcher</th>
<th>Setting/Participants</th>
<th>Methods</th>
<th>Treatment</th>
<th>Study Findings</th>
</tr>
</thead>
</table>
| Johnson & Partlo   | • Two elementary classrooms                                | Qualitative:                                                           | Exposure to 10 number talks                        | • Positive effect on students’ mental mathematics abilities and problem-solving strategies  
                                                                          | (2014)                                                     | • Student questionnaire                                       | (5 addition; 5 subtraction)                                                                                                                                  | • Growth in student knowledge of mental math strategies     |                                                                 |
|                    | • Midwestern, city                                         | • Student interviews                                                  |                                                     | • Greater accuracy when solving problems  
                                                                          |                                                             | • Teacher reflective journals and anecdotal notes             |                                                                                                                                  | • Ability to express reasoning process                      |                                                                 |
|                    | • 4th graders                                              | Quantitative:                                                          |                                                     |                                                                                                                                  |                                                                 | • Pre- and posttest design                                |                                                                                                                                  |                                                                 |
|                    |                                                             | • Pre- and posttest design                                             |                                                     |                                                                                                                                  |                                                                 |                                                                      |                                                                                                                                  |                                                                 |
| Washington         | • Medium-sized school district                              | Qualitative:                                                          | Compared scores on the STAAR test to frequency    | • Significant correlation between frequency of number talks and 6th grade STAAR scores  
                                                                          | (2015)                                                     | • teacher and administrator questionnaire                     | of number talks implementation                               | • Teachers and administrators believed number talks      |                                                                 |
|                    | • North Texas                                              | Quantitative:                                                          |                                                     | develop critical thinking skills, number sense, and problem solving  
                                                                          |                                                             | • One-way ANOVA                                             |                                                                                                                                  |                                                                 |
|                    | • 3rd-8th graders                                          |                                                                        |                                                     |                                                                                                                                  |                                                                 |                                                                      |                                                                                                                                  |                                                                 |
|                    | • 37 teachers                                              |                                                                        |                                                     |                                                                                                                                  |                                                                 |                                                                      |                                                                                                                                  |                                                                 |
|                    | • 11 administrators                                        |                                                                        |                                                     |                                                                                                                                  |                                                                 |                                                                      |                                                                                                                                  |                                                                 |
| Okamoto            | • Middle schools                                           | Qualitative:                                                          | Exposure to 15 number talks over a six-week       | • Three teacher practices to support student learning during number talks:  
                                                                          | (2015)                                                     | • Classroom videos                                         | period                                                             | (a) examine student work as a formative assessment;  
                                                                          |                                                             | • Student work samples                                      |                                                                                                                                  | (b) use visual representation; and  
                                                                          |                                                             | • Teacher journals.quantitative:                           |                                                                                                                                  | (c) focus on one idea for the  
                                                                          |                                                             | • Pre- and posttest design                                 |                                                                                                                                  | duration of the number talk  
                                                                          |                                                             |                                                                        |                                                                                                                                  |                                                                 |
|                    | • Large urban school district                              | Quantitative:                                                          |                                                     | • Statistically significant increase on number sense/ equivalent expressions scores  
                                                                          |                                                             | • Pre- and posttest design                                 |                                                                                                                                  | • Number talks effectively implemented by teacher       |                                                                 |
|                    | • 22 sixth graders                                         |                                                                        |                                                     | • Increased student engagement led to increased understanding of content.  
                                                                          |                                                             |                                                                        |                                                                                                                                  | • Increased student engagement led to increased understanding of content.  
                                                                          |                                                             |                                                                        |                                                                                                                                  | (c) focus on one idea for the duration of the number talk  
                                                                          |                                                             |                                                                        |                                                                                                                                  |                                                                 |
| Turner             | • Elementary school                                        | Qualitative:                                                          | Exposure to number talks at least 3 times per     | • Statistically significant: increase in student achievement  
                                                                          | (2017)                                                     | • Questionnaire, observation, anecdotal notes, voice recordings | week                                                                | • Number talks effectively implemented by teacher       |                                                                 |
|                    | • Georgia                                                  | Quantitative:                                                          |                                                     | • Increased student engagement led to increased understanding of content.  
                                                                          |                                                             | • Pre- and posttest design                                 |                                                                                                                                  | • Increased student engagement led to increased understanding of content.  
                                                                          |                                                             |                                                                        |                                                                                                                                  | (c) focus on one idea for the duration of the number talk  
                                                                          |                                                             |                                                                        |                                                                                                                                  |                                                                 |
|                    | • 5 second-grade ESOL students                             |                                                                        |                                                     | • Inclusion of visual representations and multiple strategies is critical in supporting problem solving.  
                                                                          |                                                             |                                                                        |                                                                                                                                  | • Inclusion of visual representations and multiple strategies is critical in supporting problem solving.  
                                                                          |                                                             |                                                                        |                                                                                                                                  | (c) focus on one idea for the duration of the number talk  
                                                                          |                                                             |                                                                        |                                                                                                                                  |                                                                 |
Overview of Empirical Research

A review of the existing empirical research on classroom number talks indicated that the majority of current studies selected focal participants at the elementary- or middle-school levels. Quantitatively, researchers have primarily concentrated on student achievement as the result of regular exposure to classroom number talks. Most quantitative studies employed a pretest-posttest design to determine the impact of number talks on students’ learning of mathematics. Research interests related to student achievement included problem solving, computation, strategies, and number sense. The findings of the research were varied, with disparities in the statistical significance of number talks and its influence on achievement.

Qualitatively, researchers have mainly focused on students as the primary units of analysis. Much of the current research has reviewed formative assessment data to determine the effects of number talks on students’ number sense, used observations to monitor student engagement, participation, and autonomy during number talks, and examined the outcome of students’ mental computation strategies as the result of classroom number talks. To date, only two qualitative studies have opted to focus on teacher participants. Washington (2015) distributed a survey to both teachers and administrators in grades 3-8 to determine their beliefs and attitudes about classroom number talks. Turner (2017) observed the implementation practices of one classroom teacher as she worked with a small group of students who qualified for ESOL services. Research on the role of number talks in developing number sense and number relationships in third-grade students pursuant to teacher perception was not evident at the
time of this study. Furthermore, no existing research had explored how teachers implement number talks in practice according to Parrish’s (2011, 2014) five fundamental components.

Summary

This chapter provided an overview of the theoretical framework that guided this study and a literature review on number sense, discourse, and classroom number talks. A review of the literature established that number sense is at the core of learning and understanding in mathematics. Students who demonstrate keen number sense can apply their knowledge of number and operations to solve problems efficiently, accurately, and flexibly (NCTM, 2000; Russell, 2000). Moreover, students who are able to explain effectively why their strategies are valid exhibit understanding of the subject (CCSSI, 2018a). The practice of meaningful discourse is essential for increasing students’ knowledge, understanding, and learning in mathematics (Franke et al., 2009).

The empirical research review on classroom number talks presented several quantitative investigations of the impact of number talks on student achievement in mathematics. An evaluation of qualitative studies on the topic revealed a focus on the instructional practices, attitudes, and beliefs of teachers and administrators. However, a lack of existing empirical research failed to show how classroom number talks support third-grade students’ reasoning and understanding of number concepts pursuant to teacher perception, as well as how teachers implement number talks in practice.
CHAPTER 3

RESEARCH DESIGN AND METHODOLOGY

This chapter offers a description of the qualitative research design selected to investigate the research questions addressed in this study. The chapter begins with a review of the problem statement and research questions, followed by an overview of the research design and rationale, participants and setting for the study, data collection and implementation, data analysis, role of the researcher, measures of dependability and credibility, and ethical safeguards.

Problem and Purpose Statement

Mathematics education in the United States has traditionally focused on repetition and memorization of basic skills and procedures with minimal opportunities for understanding (Burns, 2012; Klein, 2003; National Council of Teachers of Mathematics [NCTM 1989; National Research Council [NRC], 2001; Parrish, 2014; Sood & Mackey, 2014). The role of the teacher in U.S. classrooms has traditionally centered on relating information, offering few peer collaborative experiences to discuss and think through problem solving (Lampert, 1990). International assessment scores consistently show that students in other industrialized nations outperform U.S. students in mathematics (National Center for Education Statistics [NCES], 2016; Spring, 2011). Additionally, national assessment scores indicate that less than half of U.S. students are proficient in mathematics (National Assessment of Educational Progress, 2018).
To develop mathematical proficiency, it is essential that students engage in tasks designed to promote number sense and reasoning. However, algorithmic procedures continue to be the essence of mathematics instruction in many classrooms in the United States (Burns, 2012). Research shows that student interaction and discourse in mathematics are critical to developing meaning and understanding (Franke et al., 2009; Lampert, 1990; NCTM, 2000), yet instructional practices are typically comprised of teacher-guided lessons instead of student-centered activities that encourage sense making. A classroom environment that promotes flexibility with numbers is critical to fostering student reasoning in mathematics (Van de Walle, Karp, & Bay-Williams, 2013).

A review of the literature revealed that number sense is the heart of mathematics instruction (Boaler, 2015; NCTM, 2000; Washington, 2015). Classroom number talks are designed to cultivate the development of number sense in students through accurate, efficient, and flexible mental strategies (Parrish, 2014). However, the current literature on classroom number talks neglected to establish how this instructional practice nurtures third-grade students’ reasoning and understanding of number sense and number relationships in mathematics. This study investigated teacher perceptions on the role of number talks in supporting third-grade students with number concepts to alleviate the void in the research.

Research Questions Reiterated

The guiding question for this qualitative study was

1. What is the role of number talks in developing number sense and number relationships in third-grade students pursuant to teacher perception?
The following subquestions supported the guiding question:

a. How do teachers develop classroom community in number talks?

b. How do teachers perceive their role in classroom number talks?

c. How do mental computation strategies support students with mathematical understanding as observed by teachers?

d. How do teachers select purposeful computation problems for classroom number talks?

e. How do number talks promote student discourse in mathematics as reported by teachers?

f. What questions do teachers ask to elicit student responses during number talks?

Research Design and Rationale

This study followed a case study research design, which is a qualitative approach to research that “investigates a contemporary phenomenon in depth and in its real-world context” (Yin, 2014, p. 237). Yin (2014) acknowledged three conditions for determining the appropriateness of a case study design. The first condition concerns the research questions addressed in the study. The purpose of a case study design is to answer “how” and “why” (Yin, 2014, p. 2) questions. The second condition relates to the behavioral events of the study. In a case study design, the researcher has little or no control over the events that may transpire in the field (Yin, 2014). The third condition states the research study should concentrate on a contemporary phenomenon (Yin, 2014). Since the intent
of this research study meets these conditions, I utilized a case study design to investigate the phenomenon of the role of math talks in a third-grade classroom.

Case study research may include single or multiple cases (Yin, 2014). This study incorporated an embedded, multiple-case study design, since this design involves the study of two or more cases and includes the collection of data from “a unit lesser than the main unit of analysis” (Yin, 2014, p. 238). Yin (2014) recommended multiple-case study designs when plausible, since the results are more robust and compelling than those of single-case study designs. Moreover, Merriam (2001) claimed, “The inclusion of multiple cases is, in fact, a common strategy for enhancing the external validity or generalizability of your findings” (p. 40). In this case study, three teacher participants served as the primary units of analysis, each representing a different case. Embedded within each case were student participants, who served as the subunits of analysis.

Participants, Setting, and Sample Strategy

The teachers and school settings for this case study were identified for participation through purposeful sampling, a method in which “researchers intentionally select individuals and sites to learn or understand the central phenomenon” (Creswell, 2012, p. 206). Additionally, I conducted maximal variation sampling to acquire multiple perspectives on the role of number talks in constructing number sense and number relationships in elementary students. Creswell (2012) explained, “Maximal variation sampling is a purposeful sampling strategy in which the researcher samples cases of individuals that differ on some characteristic or trait (e.g., different age groups)” (pp. 207-208).
Although participants were selected because of convenience and accessibility to the researcher, each case met five criteria for inclusion in the study: (a) the participant was assigned to teach third-grade mathematics during the 2018-2019 school year at one of the three settings identified for the study; (b) the participant had a minimum of five years teaching experience; (c) the participant had completed a K-5 mathematics endorsement program; (d) each participant was proficient in facilitating number talks; and (e) each participant consistently facilitated classroom number talks as part of mathematics instruction. Because teachers who complete the requirements for a mathematics endorsement typically demonstrate increased content and pedagogical knowledge in the subject area, participants selected for this study were required to hold an endorsement in K-5 mathematics. Teacher proficiency in facilitating number talks was determined through conversations with both the participant and school principal. Additionally, each participant had previously attended professional learning on classroom number talks. Finally, since administrators at each of the schools selected for participation required all teachers to include classroom number talks, each participant consistently facilitated number talks as part of their instructional practice.

This research study was conducted at three different elementary schools located in a large suburban school district in the southeastern United States: Maple Elementary (pseudonym), Cedar Elementary (pseudonym), and Laurel Elementary (pseudonym). Although convenience and accessibility were factors in selecting locations, the school demographics were carefully considered to ensure the results of the study were generalizable across different populations of students. These settings represented high-
and low-income families, diverse student populations, and divergent levels of proficiency in mathematics among third-grade students.

Maple Elementary served approximately 1,000 students in grades kindergarten through five. Sixty-four percent of the students identified as White, 8% identified as Black, 10% identified as Hispanic, 15% identified as Asian, and 3% identified as Multiracial. Nearly 9.4% of the students at Maple Elementary School qualified for free-or-reduced lunch. According to 2018 assessment data, 82.6% of students in grade three scored in the proficient or distinguished learner range in mathematics.

Cedar Elementary was a Title I school that served approximately 800 students in grades kindergarten through five. Ten percent of the students identified as White, 58% identified as Black, 28% identified as Hispanic, 1% identified as Asian, and 3% identified as Multiracial. Nearly 80.77% of the students at Cedar Elementary School qualified for free-or-reduced lunch. According to 2018 assessment data, 32.7% of students in grade three scored in the proficient or distinguished learner range in mathematics.

Laurel Elementary was a Title I school that served approximately 400 students in grades kindergarten through five. Ten percent of the students identified as White, 50% identified as Black, 35% identified as Hispanic, 1% identified as Asian, and 4% identified as Multiracial. About 90% of the students at Laurel Elementary School qualified for free-or-reduced lunch. According to 2018 assessment data, 25.8% of students in grade three scored in the proficient or distinguished learner range in mathematics.
Data Collection and Instrumentation

Over the course of a six-week period, I collected multiple forms of data to determine the role of number talks in constructing number relationships and mental computation in elementary students. Yin (2014) maintained, “A major strength of case study data collection is the opportunity to use many different sources of evidence” (p. 119). Evidence from multiple sources allows for triangulation of the data, which Yin (2014) defined as “the convergence of data collected from different sources, to determine the consistency of a finding” (p. 241). Data for this case study were gathered from various sources of documentation including: (a) audio-recorded, semistructured interviews with teacher participants, (b) video-recorded sessions of classroom number talks, (c) fieldnotes from classroom observations, and (d) teacher artifacts, such as lesson plans and anchor charts. All data were transcribed, coded, and analyzed for themes. To organize and manage the amount of data collected, I used Parrish’s (2011, 2014) five tenets for effectively implementing classroom number talks as a guiding framework in coding data. These tenets consist of (a) a classroom environment that is safe for the sharing of student ideas, (b) a facilitator who questions instead of tells students, (c) mental math strategies that encourage efficient strategies, (d) purposefully selected computation problems that guide students in developing mathematical relationship and patterns, and (e) discourse that is rich in communicating mathematical knowledge (Parrish, 2011, 2014). The following sections provide a comprehensive overview of the data collection process.
Semistructured Interviews and Classroom Observations

Over a six-week period, each teacher participant completed four semistructured interviews. In a semistructured interview, the researcher develops a list of potential questions to ask the participants, but the wording and order of the questions are flexible to allow for open-ended conversation that is relevant to the moment (Merriam, 2001). To begin the research study, each participant completed an audio-recorded, introductory interview. The purpose of the initial interview was to establish each participant’s personal history, experiences, and philosophical beliefs regarding mathematics education and number talks. Prior to beginning the study, the teacher participant and I developed a six-week schedule for both interviews and classroom observations. Observations were completed during the timeframe that the classroom teacher typically conducts number talks to limit disruption of student routines. Interviews occurred during the teacher’s planning or at the conclusion of the school day.

After conducting the introductory interview, I transcribed the conversations and emailed the transcripts to each participant. I asked members to review the transcripts for accuracy and identify any items where clarifications or additional details were needed. After the findings of the study were available, I again asked each participant to review the results and provide feedback. This technique, known as member checking, increases the reliability and precision of the study when reporting the data (Lichtman, 2013).

Approximately three days after the initial interview, I visited each participant’s classroom to observe number talks in action. I video recorded and transcribed each number talks observation. Additionally, I recorded detailed fieldnotes in a journal to
document questions, thoughts, and reflections. Within three days of each observation, a follow-up interview took place to debrief the number talks session. Since each number talk is unique to the classroom community of learners, I developed interview questions after each observation based on teacher-to-student and student-to-student discourse during the session (see Appendix E). I conducted three classroom observations and three follow-up interviews throughout the six-week study period.

Artifacts

To enhance the information provided through interviews and observations, I discussed weekly lesson plans with each participant to identify patterns in instructional planning. During the interviews, the teacher participant and I conferred about former, current, and future lesson plans to determine how to select number strings for classroom number talks. Additionally, classroom anchor charts were occasionally referenced during the number talks observations to remind students of vocabulary terms and strategies for solving problems.

Data Analysis

During the six-week study period, I collected, transcribed, and analyzed the data for themes that emerged in the research. Merriam (2001) claimed, “The right way to analyze data in a qualitative study is to do it simultaneously with data collection. . . . Data that have been analyzed while being collected are both parsimonious and illuminating” (p. 162). Due to the large amount of data associated with qualitative research, I organized and coded the data in a table for ease of retrieval.
The constant comparative method for data analysis was used to identify categories and subcategories in the data. Merriam (2011) provided five guidelines for identifying categories and themes derived from the data. According to Merriam (2011), the categories should (a) "reflect the purpose of the research", (b) "be exhaustive", (c) "be mutually exclusive", (d) "be sensitizing", and (e) "be conceptually congruent" (pp. 183-184). After establishing the categories and themes for the study, I thoroughly examined the data for additional information relevant to the research.

Throughout the data collection process, I analyzed transcripts, fieldnotes, and artifacts for patterns within a single case using Parrish’s (2011, 2014) five tenets of number talks as a guiding framework. I then compared the patterns that emerged for each set of data and joined them with patterns identified in other sources of data. The patterns that recurred in the data became the categories or themes for the research (Merriam, 2011). Finally, I organized the data by theme in a table for the purpose of cross-analysis (Merriam, 2001).

Merriam (2001) identified two phases of data analysis for multiple-case study designs. The first phase, within-case analysis, involves the independent analysis of each case within the study. The second phase, cross-case analysis, connects themes that emerge across the cases in the study. When discussing cross-case analysis Merriam (2001) wrote:

The level of analysis can result in little more than a unified description across cases; it can lead to categories, themes, or typologies that conceptualize the data
from all cases; or it can result in building substantive theory offering an integrated framework covering multiple cases. (p. 195)

Since this study incorporated an embedded, multiple-case study design, I conducted both within- and cross-case analyses.

Role of the Researcher

I am a product of the public-school system during the 1980s and 1990s. My experiences as a mathematics student were traditional in that a typical class period consisted of the teacher showing and explaining a procedure, followed by student practice of numerous problems from a textbook. Although I typically received high scores in math, my knowledge and understanding of the subject were limited to regurgitating the methods of my teachers. My first struggles with the subject came at the high-school level where most of the upper-level mathematics courses focused on abstract problems and thinking. I persevered through the classes, studying the content until I was able to mimic similar problems on an assessment, but my learning was constrained to a process of skills with minimal understanding of why the mathematics worked. Later, during my years as an undergraduate student at a major university, an instructor presented content that developed conceptual understanding for the foundations of mathematics. The use of manipulatives and models to represent problems established the building blocks for sense making of why the rules and procedures I had previously learned were valid methods.

The Common Core State Standards emphasize the need for students to understand the mathematics. The standards are arranged in a progressive manner to assist students
with developing foundational skills prior to introducing standard rules and procedures. While the standards focus on a need for understanding, many teachers in classrooms across the United States continue to educate students in a traditional manner that limits the exploration and investigation of discovering mathematics. Number talks, by design, allow students to challenge their own mathematical thinking as well as the thinking of others, justify their reasoning for why strategies are efficient and appropriate, and understand the mathematics in a collaborative, nonthreatening classroom environment. During classroom number talks, students are not only communicating their own thought processes, they are also actively engaged in understanding the ideas presented by their peers. The practice of number talks concentrates on developing number sense in students by constructing reasoning through understanding. Philosophically, I believe the implementation of number talks across grade levels assists in bridging the gap from memorization to understanding in mathematics.

Dependability and Credibility

Yin (2014) acknowledged that a quality case-study design should establish dependability and credibility through four tests for social research. Yin (2014) identified these four tests as construct validity, internal validity, external validity, and reliability. Each of the four tests were applied at various stages throughout the study to increase the trustworthiness of the results.

Yin (2014) defined the first test, construct validity, as “the accuracy with which a case study’s measures reflect the concepts being studied” (p. 238). In this research investigation, multiple sources of data, including (a) audio-recorded, semistructured
interviews, (b) video-recorded sessions of classroom number talks, (c) field notes from classroom observations, and (d) teacher artifacts were collected and analyzed in order to triangulate the data. Both the audio- and video-recordings were transcribed, coded, and organized in a table, along with data obtained from field notes and teacher artifacts.

The second test, internal validity, is “the strength of a cause-effect link made by a case-study, in part determined by showing the absence of spurious relationships and the rejection of rival hypotheses” (Yin, 2014, p. 239). To ensure internal validity of this study, I asked teacher participants to review the video recordings and transcripts from the classroom number talks observation sessions and participate in debriefing interviews. Following each interview, the participants received the opportunity to review the transcripts and offer clarifications or additional details for accuracy. I also asked participants to confirm the results of the study. Furthermore, I solicited collegial collaboration in the identification of codes, categories, and themes that emerged in the data (Merriam, 2001). I developed a table to depict a comparison of the findings and how the results of the study corroborated.

The third test, external validity, as defined by Yin (2014), is “the extent to which the findings from a case study can be analytically generalized to other situations that were not part of the original study” (p. 238). Merriam (2001) described three strategies for strengthening the generalization of case study results to other situations: (a) “rich, thick description”, (b) “typicality or modal category”, and (c) “multisite designs” (pp. 211-212). To increase external validity in this study, teachers from three diverse elementary schools were selected for participation. A thick description of each participant and
school setting was depicted, including the academic background of the teacher and the demographics of the school. Furthermore, the findings of the study included a rich description of the participant’s perceptions for each research question. Last, as demonstrated in the research on constructivist teaching, teachers must be knowledgeable of the content in order to teach the subject effectively (Cobb, 1988; Greenes, 2008). As a result, teacher participants were required to meet five criteria for inclusion in the study. To reiterate, each participant (a) taught third-grade mathematics during the 2018-2019 school year at one of the three settings identified for the study; (b) had a minimum of five years teaching experience; (c) had completed a K-5 mathematics endorsement program; (d) possessed proficiency in facilitating number talks; and (e) consistently facilitated classroom number talks as part of mathematics instruction. The establishment of these parameters increased the typicality and transferability of the study’s findings to other settings where the teacher is experienced and knowledgeable of the content.

The fourth test, reliability, is “the consistency and repeatability of the research procedures used in a case study” (Yin, 2014, p. 240). This research study relied on multiple forms of evidence in order to triangulate the data and strengthen the reliability of the study (Merriam, 2001). Moreover, Merriam (2001) claimed that reliability can be increased if the researcher states in explicit detail “how the data were collected, how categories were derived, and how decisions were made throughout the inquiry” (p. 207). In this case study, the procedures for collecting, compiling, and analyzing the data were clearly defined, the process for identifying codes, categories, and themes were thoroughly discussed, and all conditions for decision making were fully disclosed. Furthermore, I
developed a chain of evidence to strengthen the trustworthiness of the research findings (see Appendix G).

Ethical Safeguards

Prior to the study commencement, I completed and filed an application for conducting research involving human subjects with the Mercer University Institutional Review Board (IRB). The approved IRB form is located in Appendix A. Following approval from the Mercer University IRB, I submitted a request to officials of the district selected for participation to conduct curriculum research. After receiving approval at the district level, I obtained permission from administrators of the schools involved in the study. Each teacher participant received an informed consent explaining the study rationale, purpose, and data collection procedures. A copy of the teacher informed consent form is located in Appendix B. Additionally, the parents or legal guardians of each third-grade student in each participating teacher’s class received a parental informed consent regarding the parameters of the study. The letter was available to parents in both English and Spanish (see Appendix C). I disbursed a student assent letter (see Appendix D) to all student participants to disclose the terms and conditions of the study. I afforded all participants and parents of participants opportunities to ask clarifying questions prior to granting permission to participate in the study, and each individual received a copy of the informed consent for personal reference.

All information obtained during the course of the study was held in strictest confidence. I assigned pseudonyms to replace the names of all participants and locations to maintain anonymity in recordings, transcripts, data analysis, and results, with all
identifiable information removed from the files. All sources of data were kept in a locked filing cabinet in my home for the duration of the study. Since I was not responsible for professionally evaluating any of the participants, there were no anticipated consequences or pressures for teachers or students to participate in the study.

Summary

This chapter presented a description of the methodology for this research study on the role of classroom number talks in constructing number concepts in third-grade students pursuant to teacher perceptions. This study utilized an embedded, multiple-case study design. Teacher participants from three diverse school settings in the southeastern United States served as the primary units of analysis for this study, and third-grade students in the participants’ classrooms served as the subunits of analysis. Data collected over the course of a six-week period included observations, interviews, fieldnotes, and artifacts. Data were analyzed using the constant comparative method. The study incorporated a multiple-case study design. Both a within-case analysis and a cross-case analysis were conducted. Chapter 4 presents the findings.
CHAPTER 4

FINDINGS

This chapter presents an analysis of the research findings associated with this study. The chapter begins with a review of the purpose of the study, along with the research questions investigated. The remainder of the chapter includes an overview of the primary units of analysis, the protocol for interviews and observations, and an examination of the data.

Purpose Statement and Research Questions Reviewed

A review of the literature indicated that number sense is a critical part of understanding mathematics (Boaler, 2015; National Council of Teachers of Mathematics [NCTM], 2000; Washington, 2015). The purpose of classroom number talks is to foster number sense and number relationships in students by developing mathematical fluency through mental computation strategies (Parrish, 2014). This study explored how number talks cultivate third-grade students’ reasoning and understanding of number concepts in mathematics pursuant to teacher perception. The following research questions guided this study:

1. What is the role of number talks in developing number sense and number relationships in third-grade students pursuant to teacher perception?

The following subquestions supported the guiding question:

a. How do teachers develop classroom community in number talks?
b. How do teachers perceive their role in classroom number talks?

c. How do mental computation strategies support students with mathematical understanding as observed by teachers?

d. How do teachers select purposeful computation problems for classroom number talks?

e. How do number talks promote student discourse in mathematics as reported by teachers?

f. What questions do teachers ask to elicit student responses during number talks?

The research findings for this study derived from an analysis of participant practices and perspectives on instructional routines during classroom number talks. Three third-grade teachers served as participants for this study. Over the course of a six-week period, each participant partook in four, audio-recorded semistructured interviews and three video-recorded number talks observations. The interviews and observations were transcribed and shared with each participant. Participants reviewed and provided feedback on each transcription. The data were then coded and organized into a table where they were analyzed for categories and themes that emerged in the set.

Participant Overview

This case study included three third-grade teachers who consistently conducted number talks as part of their mathematics instruction and served as the primary units of analysis for the research. The participants for this study taught in a large, suburban school district in the southeastern United States. In an effort to maintain the integrity of
the study, the participants were unaware of other teachers partaking in the study. Since the participants worked at different schools, the ideas presented in the case study are specific to each individual and not influenced by other teachers participating in the study. Table 5 displays the ethnicity, years of teaching experience, assigned teacher and school pseudonyms, and the percent of third grade-students who scored in the proficient and distinguished categories on the 2018 state assessment in mathematics.

Table 5

*Description of Participants*

<table>
<thead>
<tr>
<th>Participant Pseudonym</th>
<th>Ethnicity</th>
<th>Years of Teaching Experience</th>
<th>School Pseudonym</th>
<th>Percent of Grade 3 Students Proficient or Distinguished in Mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mrs. Miller</td>
<td>Caucasian</td>
<td>15</td>
<td>Maple Elementary</td>
<td>82.6%</td>
</tr>
<tr>
<td>Mrs. Addison</td>
<td>African American</td>
<td>6</td>
<td>Cedar Elementary</td>
<td>32.7%</td>
</tr>
<tr>
<td>Mrs. Knight</td>
<td>African American</td>
<td>17</td>
<td>Laurel Elementary</td>
<td>25.8%</td>
</tr>
</tbody>
</table>

Mrs. Miller

Mrs. Miller (pseudonym) was a Caucasian, female teacher of third-grade students at Maple Elementary School who had taught elementary students for 15 years. She held an Educational Specialist Degree and endorsements in teacher leadership and K-5 mathematics.
Mrs. Addison

Mrs. Addison (pseudonym) was an African American, female teacher of third-grade students at Cedar Elementary School who had taught elementary students for six years. She held a Bachelor of the Arts degree and endorsements in English to Speakers of Other Languages (ESOL), K-5 mathematics, and gifted education.

Mrs. Knight

Mrs. Knight (pseudonym) was an African American, female teacher of third-grade students at Laurel Elementary School who had taught elementary students for 17 years. She held a Master of Arts degree and an endorsement in K-5 mathematics.

Interview and Observation Protocol

Prior to the start of the study, each teacher participant provided her daily schedule for conducting classroom number talks. Additionally, participants supplied a timeframe that was convenient for completing the initial interview and the debriefing interviews. In a collaborative effort between each participant and the researcher, a series of observations and interviews were scheduled for the duration of the six-week study period. I conducted observations during the timeframe that each participant regularly devoted to number talks to limit disruption of classroom routines. All interviews took place either during the teacher’s planning period or at the end of the school day.

Because the classroom teacher and I had collaboratively developed a calendar for observations, the student participants were aware of my scheduled visits. To maintain consistency in the classroom environment, I remained in the back of the classroom until the teacher indicated she was ready to begin number talks. Once the teacher
acknowledged that she was ready to begin, I positioned the video recorder behind the students to avoid interrupting the learning community. Although classroom number talks typically lasted between 5 to 15 minutes, most of the observations for this study continued for approximately 20 minutes. Within three days of each observation, the teacher and I met to debrief the session. During each debriefing, the teacher reflected on pedagogical practices; offered insight into student conversations, strategies, and misconceptions; and received feedback on instructional approaches.

Participant Experiences and Perspectives in Mathematics

Prior to the first observation, each participant completed an initial interview. The beginning of the interview focused on each participant’s personal experiences as a student in mathematics in order to gain a better perspective on how previous occurrences may have impacted current teaching practices. At the start of the initial interview, I asked each participant to share a personal experience as a math student and how that experience has impacted her own teaching practices.

Mrs. Miller stated, “I was not good at memorizing my math facts.”

I asked, “Does that experience impact the way you teach math to your third-grade students?”

Mrs. Miller replied, “It does, because I don’t remember explicitly being taught multiplication facts. It was very procedural. Now, it’s very conceptual, and my students have strategies for figuring out [the math].”

Mrs. Addison revealed, “We did a lot of memorization and a lot of tricks.”

I asked, “How has that experience impacted your teaching?”
Mrs. Addison responded, “Looking back, I didn’t really know the connections between [mathematical concepts]. Because I now understand how things are related, I can better explain [the concepts] and help my students.”

Mrs. Knight stated,

I was never intimidated by math, but it was not my favorite subject. It was just something that I did not mind doing. As I went further and further up, like into my later high school years, it was a struggle just to stay on course and get through the classes.

I asked, “In what ways has that experience impacted how you teach math today?”

Mrs. Knight replied,

I think that, over time, the world of education has changed. I started teaching in 1996, so it was really fact-based, practice, and drills. At that time, the most successful students were the ones who could memorize the most and apply that to different scenarios quickly. It was really about just being correct. Now, I’m always trying to dig into “What are you thinking? Why are you applying that [strategy]? Tell me more about what you’re thinking.”

These exchanges established that all three participants had similar experiences as a student of mathematics: an overreliance on memorizing skills and a lack of connections and relationships among concepts. As a result of these experiences, each participant noted how their third-grade students now benefit from learning mathematics in a conceptual manner, while exploring the relationships that exist between numbers.
Organization of Data Analysis

This research study incorporated an embedded, multiple-case study design. Merriam (2001) identified two phases of data analysis for this type of research: a within-case analysis and a cross-case analysis. First, I conducted a within-case analysis of each individual case. The findings of the within-case analyses are discussed in the following sections. Later in the chapter, I examine the themes that emerged in the cross-case analysis.

Verbatim quotations are sometimes difficult to comprehend because of the pauses, repetition, and incomplete thoughts that occur in everyday speech. Corden and Sainsbury (2006) conducted a study on the views of qualitative researchers in using verbatim quotations to report research findings and found that “to enhance readability, some researchers expected to do some re-punctuation. It was also common practice to take out the ‘ums’ and ‘ers,’ phrases such as ‘I mean’ and ‘you know,’ and the word repetitions which pepper most people’s speech” (p. 18).

Furthermore, Corden and Sainsbury (2006) wrote:

One person said it seemed patronizing to reproduce hesitancies and false starts in normal speech, which told the reader nothing except that the speaker was taking some time to think or needed to practice what they wanted to say. Another felt even more strongly that reproducing the hesitancies in some people’s speech did them a disservice, because of negative judgments which readers might make about speakers. (p. 18)
Based on these findings and in an effort to increase readability and understanding, I did a “light tidying-up” (Corden & Sainsbury, 2006, p. 18) of the dialogue that occurred during both the observations and the interviews.

Trustworthiness of Results

Yin (2014) recommended a series of four tests to strengthen the trustworthiness and rigor of case-study research: construct validity, internal validity, external validity, and reliability. During the research study period, I collected, transcribed, coded, and analyzed multiple forms of data for triangulation and construct validity. Data reviewed for this study included audio-recorded interviews, video recordings of classroom number talks, field notes, and teacher artifacts.

To maintain internal validity, I reviewed each video recording prior to the debriefing interview and developed clarifying questions to enrich my understanding of the data. Each participant reviewed transcriptions of the audio and video recordings and provided modifications as needed. I assigned a priori codes and organized the data in a table for further analysis. For consistency, I enlisted the collaborative support of a colleague to identify codes, categories, and themes in the data. After reviewing and identifying themes individually, a joint conversation ensued to determine the appropriate phrasing to portray the data. A table depicting a comparison of the themes is located in Appendix F. After the results were available for review, each participant examined the findings and granted permission for publication.

To ascertain external validity and generalizability of the study, I initiated three safeguards. First, the settings for this study consisted of three different schools with
diverse student populations. Second, the findings of the study included a thick, rich description of the participant’s perspectives for each research question, which were supported with data collected during the classroom observations. Third, each participant selected for this study had a minimum of five years teaching experience and held an endorsement in K-5 mathematics.

Finally, to ensure the findings were reliable, I coded and categorized the data using consistent measures. During the first cycle of coding, the a priori codes of number sense, number relationships, classroom community, the teacher’s role, mental math, purposeful computation problems, classroom discussions, and questioning were generated to disaggregate the data. I then subjected the data to a second cycle of coding to determine the most frequent codes in the set. The codes were later condensed into categories and themes. In the final review of the data set, I examined the categories for themes that emerged across the cases. The resulting themes were (a) verbalizing reasoning, (b) increased language and communication skills through class discussions, (c) kind, respectful learning environment, (d) facilitator who selects purposeful number strings, and (e) student-to-teacher discourse.

Within-Case Analysis

During the first phase of data analysis, I conducted a within-case analysis, which involved the independent investigation of each case within the study (Merriam, 2001). The guiding research question for this investigation highlighted the role of number talks in developing number sense and number relationships in third-grade students pursuant to teacher perception. I analyzed data from interviews, observations, and fieldnotes using
two a priori codes that directly emerged from the research question: number sense and number relationships. The ensuing sections offer discussions of the findings.

Role of Number Talks in Developing Number Sense

Parrish (2014) claimed that by design, number talks foster number sense and reasoning in students. At the start of this study, I asked each teacher participant to reflect on the role of number talks and its influence on students’ understanding of number concepts. Following each classroom observation, I conducted an analysis of students’ responses to the problems presented in each number talk session, and during each debriefing, the classroom teacher offered her perceptions regarding student thinking and understanding of the mathematics involved with the problems presented during the session.

Mrs. Miller: Number talks and number sense. During the initial interview, I asked Mrs. Miller to reflect on the role of number talks and relate her perception of how the practice had impacted students’ understanding of number concepts. I inquired, “Reflecting on your experiences, how have number talks impacted your students’ understanding of number concepts?”

Mrs. Miller stated,

This year, I have used number talks to preview upcoming content, and it’s really had a positive impact on my [students’ understanding]. Concepts that took a while in past years [for students] to grasp are, all of a sudden, not as difficult for this group. These students have a little bit of preknowledge going into a new concept because they’ve had a chance to discuss and talk about it prior to doing
[written] work. I feel like number talks takes the pressure off the class because I’m not expecting them to produce anything or turn something in. It gives them a little bit more of an understanding going into a new concept, almost like a jumping board.

During the first number talks observation for this study, Mrs. Miller introduced the concept of fractions on a number line to students. Fractions were a new concept for this group of students, as the Common Core State Standards (CCSS) include fraction content for the first time at the third-grade level. At the start of the number talks session, Mrs. Miller displayed Figure 1 on the board and posed the following question: “What fraction is located at the point of the star on the number line? Explain your reasoning.”

Figure 1. Locating fractions on a number line.

After a wait time of approximately 20 seconds, Mrs. Miller asked various students in the classroom to state their answer to the question. Each solution was recorded on the board along with the students’ initials. Solutions to the problem included 3/8, 4/9, 3/7, and 9/3. Once the range of solutions were written on the board, Mrs. Miller asked the students who shared responses to the problem to explain their reasoning. She asked, “James, tell us why you think it’s 3/8. What was your thinking?”

James reasoned, “I don’t think you count the lines.”

Mrs. Miller responded while pointing to the lines on the figure, “Alright, so you’re not going to count the lines? So, what would you count?”
James remarked, “I would count bumps.”

Mrs. Miller stated, “Okay, help me count while I draw bumps.” James counted aloud while Mrs. Miller drew marks as shown in Figure 2 to represent the number of segments between zero and one on the number line.

James counted, “one, two, three, four, five, six, seven, eight.”

![Figure 2. Fractional parts on a number line.](image)

Mrs. Miller asked, “We had eight bumps between zero and one on the number line. What does that mean?”

James replied, “It means eighths.”

Mrs. Miller further inquired, “Does eighths represent the numerator or the denominator?”

James answered, “It’s the denominator. There are eight parts in the whole.”

Mrs. Miller clarified for the class, “So James is saying that each segment is 1/8 of the whole. The star on the number line is located after the third segment. James, what fraction is located at the point of the star on the number line?”

James announced, “3/8.”

Mrs. Miller then asked another student to share his thinking. Earlier, Liam claimed his solution to the problem was 4/9. Mrs. Miller asked, “Liam, can you explain how you got ninths?”
Liam explained, “I counted the lines instead of the spaces between the lines. There are nine lines, and the star is above the fourth line, so 4/9.”

Mrs. Miller then asked Izzy, who came up with a solution of 3/7, to explain her reasoning. Mrs. Miller inquired, “Izzy, how did you get sevenths?”

Izzy answered,

I counted the lines, but I didn’t count the lines at the zero or the one. I only counted the lines between zero and one. There are seven lines between the numbers, and the star is at the third line. That’s how I got 3/7.

Mrs. Miller then asked Russell, who earlier stated the solution was 9/3, to explain his reasoning: “Russell, how did you get thirds in the denominator?”

Russell responded, “I counted wrong with the numbers.”

Mrs. Miller questioned further, “Alright, can you tell me what you mean by that?”

Russell said, “I flipped the numbers. I was thinking 3/9. I counted all of the lines and found 9 for the bottom number. I want to change my answer to 3/8. I should have counted the spaces between the lines.”

After all students who provided solutions to the problem explained the reasoning for their response, Mrs. Miller addressed the class to confirm the correct answer. Mrs. Miller, validating James’s response, stated, “To find the number of fractional pieces in the whole, we count the spaces or bumps on the number line. There are eight bumps, or pieces, in the whole.”

Mrs. Miller labeled the number line in eighths, as demonstrated in Figure 3, to demonstrate that each section is equal to 1/8 of the whole. Mrs. Miller then asked,
“Alright, so are we all in agreement with James that the star is located at 3/8?” The students nodded or indicated with a hand signal that they agreed with James’s solution of 3/8.

![Figure 3. Fractions on a number line.](image)

The following day, Mrs. Miller and I debriefed the number talks session, and I inquired about her perception of students’ number sense understanding with fraction concepts. I asked, “Have you noticed a change in students’ learning of fraction concepts over the past few days as a result of number talks?”

Mrs. Miller replied,

Absolutely! Looking back and reflecting on prior years, this class seems to be understanding [fraction concepts] far more quickly than previous classes. I think it’s because of the previews during number talks that they’re getting the concepts more quickly. Do they still make mistakes? Yes, they still make mistakes, and they still have misunderstandings. I still have to pull students for small-group instruction during guided math, but all in all, I think they have a better understanding of the content than my former students. And some of the students in this class really lack a strong foundation of number sense, but they are still grasping the concept of fractions, and I believe it’s because of number talks.
In the interviews conducted for this study, Mrs. Miller consistently commented on the role of number talks and its impact on students’ number sense development in mathematics. At one point, I inquired about Mrs. Miller’s philosophical beliefs for incorporating number talks regularly in her classroom instruction. I asked, “Why do you include number talks in your mathematics instruction?”

Mrs. Miller explained,

I believe in number talks because I’ve seen what it can do for students. The first few years I incorporated number talks, I had a group of students who qualified for ESOL services in my class, and one of the things that I noticed was that they’re communication really improved; their speaking, communication, reasoning, and vocabulary all improved because of number talks. Students are able to think and listen to their peers without the pressure of writing down an answer. In number talks, we’re just talking and reasoning about problems.

Mrs. Addison: Number talks and number sense. At the beginning of this study, I asked Mrs. Addison to share her perceptions on the role of number talks and its influence on students’ number sense. During the first interview, she was skeptical that number talks were directly responsible for her students’ development of number. I asked, “Reflecting on your experiences, how have number talks impacted your students’ understanding of number concepts?”

Mrs. Addison declared,

I don’t know if it’s so much the number talks. It could be the number talks, but I think it’s because I give them so many different strategies to use prior to number
talks that is helping with their number sense. I don’t require them to use any one strategy. They choose which strategies work for them. When they figure problems out for themselves or use a strategy of their choice, I think that helps develop their concept of number.

I inquired further, “Why do you include number talks in your classroom?”

Mrs. Addison explained,

Number talks give all students an opportunity to show what they know. They like to show their strategies, and I get to see how different kids are solving the problems. It gives other students a chance to see the different strategies too, which is not always possible when working in small groups. Kids are sometimes better at showing and helping each other understand how to solve problems than the teacher, and number talks provide that opportunity to students.

Although Mrs. Addison expressed that number talks were an essential part of instruction for sharing strategies, she did not immediately acknowledge their impact on developing students’ number sense. However, Mrs. Addison’s perception shifted as the study progressed. During each interview, she and I discussed multiple possibilities for introducing different types of problems into the number talks session. The following vignette is a result of our conversations.

Mrs. Addison displayed the equation $18 \times 4 = 22$ on the board, and asked the class, “Is this equation true or false? Turn and talk with a partner. Decide if it’s true or false and tell me why.”
As the students discussed the problem, Mrs. Addison began to write the names of students on the board who indicated they were ready to discuss the problem.

Mrs. Addison asked, “Hayden and Miguel, what did you come up with as your solution and why?”

Miguel responded, “We said it’s false.”

Mrs. Addison restated, “It’s false? Why?”

Hayden, chiming into the conversation, stated,

Yeah, it’s false because 18 x 4 = 72. If you think about 18 x 2, or 18 + 18, that equals 36. That’s already more than 22, and you have to add another pair of 18 + 18 to get the answer for 18 x 4. If 18 + 18 = 36, then you have to double it to get 18 four times, so 36 + 36 = 72. That means 18 x 4 = 72.

Mrs. Addison, addressing the class, asked, “Do we agree or disagree with Hayden and Miguel?”

Nia asserted, “I respectfully agree, but I had a different strategy.”

Mrs. Addison replied, “Okay, Nia, what did you do?”

Nia explained, “I broke down the 18 into 8 and 10. And 10 x 4 = 40, and then 8 x 4 = 32. Then I added 40 + 32, and I got 72.”

Mrs. Addison asked, “So you used the distributive property?”

Nia responded, “Yes.”

In a follow-up interview, Mrs. Addison noted that, because of incorporating different types of problems, her perception of number talks and their role in developing
number sense in students had shifted. I inquired, “Have number talks influenced your students’ development of number concepts?”

Mrs. Addison stated,

For some students, I think they benefit from the repeated practice of using strategies to solve a problem, and that usually comes during our regular math instruction time. But as far as the verbalizing and reasoning about problems, I do think that’s coming from the number talks. Once they get the confidence with saying it aloud, they feel more comfortable with writing it down on paper.

In a later interview, I asked Mrs. Addison to share her feedback on the different types of problems that she had recently tried with students in number talks. I said, “Over the last six weeks, you have tried implementing true and false problems and turn and talks during number talks. How has that changed your perception of number talks and its impact on developing student reasoning and number sense?”

Mrs. Addison revealed,

I like doing these kinds of problems. It’s engaging and provides more opportunity for students to participate. Before, the problems were straight computation, and the same students wanted to share their solutions and strategies for solving the problems. I never incorporated the turn and talks prior to this study, and I think it’s a good approach for getting everyone engaged in the conversation. I see more of [the students] getting involved and talking instead of being afraid to share their answer. With the turn and talk, they have a chance to reason through their answer and receive peer feedback before sharing it with the entire class.
Mrs. Knight: Number talks and number sense. During the initial interview, I asked Mrs. Knight to reflect on the role of number talks and the impact of the practice on developing students’ number sense according to her perception.

I inquired, “Reflecting on your experiences, how have number talks impacted your students’ understanding of number concepts?”

In responding to this question, Mrs. Knight first recalled her experiences as a second-grade teacher where she felt that she had not implemented number talks often enough. She recalled,

I would look at other students who had been doing number talks for a while, and I could just see the difference. They were thinking about numbers in a way that my second-graders were not able to achieve. And honestly, I was a little embarrassed. So, when I moved to third grade last year, and especially because I had some of the same students from second-grade, I realized that it was time to really push myself to incorporate number talks.

I inquired further, “And have you noticed any changes in your students’ understanding of number concepts after regular implementation of number talks with your third-grade students?”

Mrs. Knight responded,

I have. They are able to fall back on the language that they use in number talks to make sense of problems and talk their way through solving problems. I have several students in this class who are English language learners, and when we’re
talking about math outside of number talks, they are using language from number talks. So, I know it benefits them.

Over the course of this study, two-thirds of the number talks observations in Mrs. Knight’s classroom focused on doubling and halving numbers. The number talks sessions were developed sequentially in order to increase students’ conceptual understanding of number concepts. Prior to this number talks session, students had been exposed to the concept of doubling, but their experiences with halving were minimal. To introduce halving, Mrs. Knight relied on students’ prior knowledge of array models for multiplication. By using representations, students were able to visualize the mathematics.

Mrs. Knight began the conversation by placing the number eight on the board. She then asked, “Callie, tell us what you would do to halve this number.”

Callie replied, “Half of eight is four, so I did $4 + 4$.”

Mrs. Knight questioned further, “Is there something you did to come up with that answer?”

Callie stated, “I thought of four columns and four rows.”

Mrs. Knight drew a model of Callie’s response as shown in Figure 4.

![Figure 4. Callie’s representation for half of eight.](image)
Mrs. Knight asked, “Do you all agree with Callie’s strategy?” After a brief pause she continued, “I see some disagreement in the room. Jackson, do you want to help Callie with her strategy?”

Jackson said, “You have to draw four rows and two columns or four columns and two rows because the total area of the inside has to be eight.”

Mrs. Knight drew a 4 x 2 array and a 2 x 4 array as shown in Figure 5.

Figure 5. Jackson’s representation for half of eight.

Mrs. Knight then clarified, “Can you tell us why these arrays represent half of eight?”

Jackson replied,

I know that 4 + 4 = 8, and that is the same as 4 x 2. That means that the array would either be four rows and two columns or four columns and two rows. The area has to be the total, which is eight.
Mrs. Knight reiterated for the class while referencing the visual models:

So, Jackson said you would either have four columns and two rows, or four rows and two columns in order to have a total of eight altogether. He said the whole area would have to be eight. Each of these models show that four is half of the whole.

Mrs. Knight then said, “Let’s take a look at Callie’s array. Callie had four rows and four columns. What does her model represent? Myra?”

Myra explained, “Callie’s model shows four groups of four, which is 16. Her model shows eight doubled instead of eight halved.”

In the debriefing interview for this observation, I asked Mrs. Knight to share her reflections on the session and how it supported students’ development of number sense.

Mrs. Knight reflected,

Doubling and halving is such a powerful skill, and if [students] understand it, they can apply it to other things like fact fluency. If students understand how to double, they can use that skill to find the products of their twos, fours, and eights facts. I’ve tried to connect the concept of halving to some of our work with fractions, and I see them making the connection just from the language perspective. I really see the results of number talks in the language and the vocabulary they use in their explanations. Because of our conversations, they have internalized the meanings of vocabulary words. They are making sense of problems, and they are writing and giving examples just from the discussions we’ve had during number talks.
Figure 6 displays the participant themes for number sense.

Role of Number Talks in Developing Number Relationships

Parrish (2014) argued that strategic number strings “elicit specific strategies that focus on number relationships and number theory” (p. 5). Likewise, Van de Walle, Karp, and Bay-Williams (2013) endorsed a learning environment where students have opportunities to flexibly grapple with number relationships, especially in mental capacities. Over the course of a six-week period, I observed each participant implementing classroom number talks three times. An analysis of each observation afforded insight into students’ thinking about numbers and the relationships between numbers. The teacher participants offered additional viewpoints and clarifications during the debriefing interviews.

Mrs. Miller: Number talks and number relationships. During the second number talks observation in Mrs. Miller’s classroom, the focus of the discussion centered on
equivalent fractions and the relationship between the numerator and the denominator.

Mrs. Miller displayed the fraction $\frac{3}{6}$ on the board and asked students to name at least one fraction that was equivalent to the number. After a wait period of about 30 seconds, Mrs. Miller began to record student solutions on the board, and the following conversation ensued.

Mrs. Miller asked, “James, do you think you have a fraction that is equivalent to $\frac{3}{6}$?”

James answered, “Yes, $\frac{12}{24}$.”

Mrs. Miller recorded James’s solution and then asked, “Amanda, do you have another equivalent fraction that you thought of?”

Amanda responded, “$\frac{1}{2}$.”

Again, Mrs. Miller recorded the answer on the board, and then asked, “Sarah, what equivalent fraction did you think of?”

Sarah replied, “$\frac{6}{12}$.”

At this point, $\frac{1}{2} = \frac{3}{6} = \frac{6}{12} = \frac{12}{24}$ was displayed on the board for students.

Mrs. Miller questioned, “Who can tell me how you know that these fractions are equivalent? Izzy?”

Izzy confidently stated, “Because three is half of six.”

Mrs. Miller repeated, “Okay, so when you looked at the numerator and the denominator, you noticed that three was half of six. Does that same relationship exist with $\frac{1}{2}$? Amanda, what do you think?”

Amanda said, “Yes, because one is half of two.”
Mrs. Miller, now addressing all of the students, asked, “So three is half of six and one is half of two. Is six half of twelve?”

The class responded as a chorus, “Yes!”

Mrs. Miller proceeds to the next fraction, “Is 12 half of 24?”

The class responded, “Yes!”

Mrs. Miller, reiterating the relationship between the numerator and the denominator, confirmed,

So, when the same relationship exists between the numerator and the denominator, the fractions are equivalent. Now let’s try a different fraction because I want to see if you notice another relationship. Can you find at least one other fraction that is equivalent to 1/3?

After a wait time of approximately 20 seconds, Mrs. Miller called on Sally to state an equivalent fraction to 1/3.

Mrs. Miller inquired, “Sally, what fraction do you think is equivalent to 1/3?”

Sally responded, “3/6.”

Mrs. Miller, referencing the hand signals of the group, observed, “I see some disagreement with Sally’s answer. Let’s talk about the relationship between the numerator and the denominator. What’s the relationship between the one and the three in 1/3?”

Mrs. Miller paused for students to think about the relationship between the numbers. When asking the question for a second time, no one was able to state the
relationship between the numerator and denominator. As a result, Mrs. Miller asked the class to consider the relationship between Sally’s answer of 3/6.

Mrs. Miller inquired, “Let’s look at Sally’s answer of 3/6. What’s the relationship between the three and the six, James?”

James replied, “Three is half of six.”

Mrs. Miller repeated, “Three is half of six. Now look at the relationship between the one and the three in 1/3. Is one half of three?”

Together, the class responded, “No!”

Mrs. Miller clarified, “No, it does not have the same relationship, so that’s why 3/6 can’t be equivalent to 1/3. Kayla, do you think you have a fraction that is equivalent to 1/3?”

Kayla answered, “1/100.”

As the conversation unraveled, Mrs. Miller recognized that students were struggling with the concept of identifying fractions that were equivalent to 1/3. Consequently, the conversation shifted to a discussion of inequalities.

Mrs. Miller remarked, “Let’s look at a visual representation of 1/3 and 1/100.”

Mrs. Miller displayed two, same-sized, circular figures partitioned into equal parts on the board. One circle represented 1/100, and the other circle represented 1/3.

Mrs. Miller stated, “Let’s look at these two figures. This circle shows 1/100 and this circle shows 1/3. Are these two figures have an equal amount of shading?”

The students, responding as a group, stated, “No!”
Mrs. Miller continued, “If I had to put a greater than, a less than, or an equal sign between 1/100 and 1/3, which sign would I put, Sam?”

Sam responded, “You would put a less than sign between the fractions.”

Mrs. Miller, pressing for more understanding, asked, “Why is 1/100 less than 1/3?”

Sam reasoned, “Because the pieces in the 1/100 circle are small. It takes 100 pieces to make up the whole circle.”

Mrs. Miller questioned further, “Okay, so what do you notice about the denominators in the fractions 1/100 and 1/3, Russell?”

Russell maintained, “I noticed that the smaller the number, the bigger the pieces. And the bigger the number, the smaller the pieces.”

Mrs. Miller corroborated, “Ah-hah! Did everyone hear what he said? He said the larger the denominator, the smaller the pieces. And the smaller the denominator, the larger the pieces. Will that logic work every time?”

As a group, the students responded with hesitations, “Yes?”

Mrs. Miller insisted, “Let’s try a few more fraction comparisons to find out.”

In the debriefing interview following this observation, I had an opportunity to gain further perspective on Mrs. Miller’s perceptions on the role of number talks in developing number relationships.

I asked, “What role have number talks played in developing students’ understanding of number relationships?”

Mrs. Miller replied,
Number talks are all about patterns and looking for number relationships. Identifying patterns and relationships among and between numbers is really important to developing a strong foundation in mathematics. At the start of the number talk, [my students] did really well with finding equivalent fractions for 1/2 because they know their multiplication facts with factors of two. It really does depend on how well they have internalized their multiplication facts. They did not do as well with finding equivalent fractions for 1/3 since most of them are not as confident with their three and six facts.

I further inquired, “This was the first time that students had experienced fraction inequalities. What role did number talks play in developing understanding of fractional relationships?”

Mrs. Miller remarked,

In third grade, one of the basic fraction concepts that students need to understand is that as the denominator gets larger, the size of the fractional pieces gets smaller. At least one student observed that the magnitude of the denominator effects the size of the pieces. As we looked at additional examples, more students began to see the pattern. By introducing this concept in number talks, students had an opportunity to form their own conclusions by examining the visuals and the denominators. They were able to observe the relationships and talk through the problem without the added pressure of doing written work.

Mrs. Addison: Number talks and number relationships. After observing number talks in Mrs. Addison’s classroom over the six-week study period, it was discernable that
students in her class struggled with place value understanding, regrouping, and number relationships. An example of this was evident during the second observation, as a student was unsuccessful in her attempt to explain her strategy for finding the product of 12 x 4. The following scenario illustrates the student’s thought process for solving the problem.

Mrs. Addison asked, “Alice, would you like to share your strategy?”

Alice replied, “Sure.”

Mrs. Addison stated, “Okay, tell me what you did.”

Alice explained, “I added 2 + 2 + 2 + 2 and got 8, and then I added 1 + 1 + 1 + 1 and got 4. So the answer to the problem is 48.”

Mrs. Addison recorded Alice’s strategy on the board, and asked her to elaborate upon her strategy for finding the product of 48.

Mrs. Addison inquired, “Alice, can you explain what you did?”

Alice remarked, “I can’t remember. That’s how I got my answer.”

As the session continued, other students shared strategies for finding the product of 12 x 4, but further investigation into Alice’s thought process was not explored. During the debriefing interview, I asked Mrs. Addison to revisit Alice’s strategy and reflect on the misconceptions in her understanding.

I asked, “What are your perceptions about the strategy that Alice shared?”

Mrs. Addison stated,

Well, when I first looked at the strategy, I thought she decomposed the 12 into 10 and 2. She calculated 4 x 2, by doing repeated addition, 2 + 2 + 2 + 2, and got 8.
But then, she added $1 + 1 + 1 + 1$, which led me to believe that she had mistaken the one for the tens place.

As the interview continued, Mrs. Addison decided to display the problem again, only this time as an error analysis. Thus, in the following week, Mrs. Addison presented the problem of $12 \times 4$ again, but this time she asked them to conduct an error analysis while discussing the problem with a partner. As the students discussed the problem, Mrs. Addison listened to the conversations and challenged students’ thinking with questions such as, “Why would you just add zeros to the ones?” or “Tell me why the ones should be 10s.” After the brief discussion time, the class revisited the problem together.

Mrs. Addison asked, “Haley, how can we correct this strategy?”

Haley said, “Instead of adding $1 + 1 + 1 + 1$, you should add $10 + 10 + 10 + 10$ because in the number 12, the one is in the tens place.”

Mrs. Addison questioned, “Okay, tell me a little more about your thinking.”

Haley replied, “If you decompose the 12 into $10 + 2$, you can multiply $4 \times 2$, or $2 + 2 + 2 + 2$, and get 8. Then, you can multiply $4 \times 10$, or $10 + 10 + 10 + 10$, equals 40. $40 + 8 = 48$.”

Ben added, “I think she might have done the problem up and down, and that’s how she got $1 + 1 + 1 + 1$.”

Mrs. Addison inquired, “Ben, tell me a little more about what you’re thinking. Would it help if I write the problem vertically?”

Ben, nodding in agreement said,
If you multiply $4 \times 2$, that equals 8. That’s the same thing as $2 + 2 + 2 + 2$. You write 8 in the ones position. Then, if you multiply $4 \times 1$, that’s 4. That’s where the $1 + 1 + 1 + 1$ came from. You write the 4 in the tens place.

Mrs. Addison clarified, “Okay, so you’re talking about the standard algorithm.”

Mrs. Addison, seizing the moment to reemphasize the meaning of place value relationships, stated,

In the number 12, the two is in the ones position and the one is in the tens position. Even with the standard algorithm, place value still applies. We need to think of this as $4 \times 2$ and $4 \times 10$, not $4 \times 1$ since the one is in the tens position, and its value is 10.

During the final debrief, Mrs. Addison and I revisited the problem once again. I asked, “What role did number talks play in clarifying student misconceptions of place value relationships?”

Mrs. Addison explained,

I was pleased that this time when they looked at [the strategy], they could automatically see the misconception. When they saw the strategy the other day, they didn’t understand, but this time, they recognized [the error] almost immediately. I think the turn and talk helped with that understanding. By talking about the problem, they were able to see that the one should have been a ten because of place value.

Another example of student struggles with number relationships occurred during the third observation. After revisiting the place value error analysis problem, Mrs.
Addison displayed the problem 267 – 89 on the board and asked students to estimate the difference. After a wait time of approximately 20 seconds, Mrs. Addison called on Miguel to share his solution to the problem.

Mrs. Addison said, “Miguel, tell me one way we could estimate this problem.”

Miguel responded, “You could estimate the problem by hundreds.”

Mrs. Addison questioned, “Okay, so if I estimate by hundreds, what would I do?”

Miguel replied, “That would be 300 + 100 = 400.”

Mrs. Addison asked the class, “Do you agree with Miguel?”

Together, the class responded, “No!”

Although Miguel was able to round both the minuend and the subtrahend to the nearest hundred correctly, he incorrectly stated that the operation was addition instead of subtraction. After the class disagreed with Miguel’s solution, no further discussion of the strategy took place. Mrs. Addison then called on Allie to share her solution to the problem.

Mrs. Addison stated, “Allie, tell me your thinking.”

Allie began, “It’s 200 – 0 and that equals 200.”

Mrs. Addison questioned, “Are you using estimation or are you solving the problem?”

Allie replied, “I’m solving it and then estimating.”

Mrs. Addison responded, “Ok, go ahead with how you solved it.”

Allie continued, “You can’t do 60 – 80 so you have to switch it around. So 80 – 60 equals 20. And then you can’t do 7 – 9. You have to 9 – 7 = 2.”
Mrs. Addison asked, “What does that equal?”

Allie, pausing to find the difference, stated, “182.”

Allie’s understanding of number relationships led her to simply reverse the numbers in the tens position and the ones position to give a positive difference. Then she calculated \(200 - 20 + 2\) equaled 182. In actuality, Allie’s strategy, known as partial differences, would have been successful, but she needed to subtract two instead of adding two. For example, she should have calculated \(200 - 20 - 2 = 178\). In Allie’s explanation, it was not clear that she understood the strategy of partial differences. Instead, it appeared she relied on the idea that when subtracting two numbers, the larger number must come first, without regard for number values and relationships. As the conversation continued, Mrs. Addison asked another student to share his solution.

Mrs. Addison inquired, “Do we agree with Allie?”

Andre said, “No, she’s off by four.”

Mrs. Addison said, “Okay, tell me how you found the difference.”

Andre responded, “I started at the 200, and made that a one.”

Mrs. Addison clarified, “You made the two a one?”

Andre replied, “Yes.”

Mrs. Addison questioned further, “And then what?”

Andre continued, “And then I made the six a 16.”

Mrs. Addison, recording the student’s strategy on the board, said, “Okay, go ahead.”
Andre, pausing for a moment, said, “So, $16 - 8 = 8$,” and after another short pause said, “I’m confused.”

In the debriefing interview following this observation, I asked Mrs. Addison to share her observations about students’ understanding of number relationships.

I asked, “Have students struggled with rounding and regrouping only in number talks or also in your regular math class?”

Mrs. Addison explained,

It’s been a struggle in my regular math block, too. Rounding, regrouping, and written expression are the three biggest struggles for my students. I find that they do not want to regroup, or they get confused when they do. A lot of them depend on reversing the numbers instead of regrouping to rename the number. For example, if [the problem] was $58 - 19$, many of them would subtract $9 - 8$ instead of regrouping or renaming the $58$ as $40$ and $18$. They have difficulties with seeing that $58$ can be renamed as $40$ and $18$. They don’t understand the relationship between the numbers.

I inquired, “Have you modeled regrouping with base-ten blocks during math class?”

Mrs. Addison replied, “Yes, we’ve modeled problems with base-ten blocks and number lines.

I asked, “How do they respond when the manipulatives are in front of them?”

Mrs. Addison remarked, “They do much better when the manipulatives are in front of them because they can concretely regroup to show the different relationships.”
I further inquired,

You mentioned that you had used number lines for subtraction, but I noticed that none of the students relied on the adding-up strategy to find the difference. What have you observed about students using friendly number relationships to find the solution?

Mrs. Addison responded,

A lot of them are able to do the [adding-up] strategy, but for some reason they don’t want to do the number line. Going back to the example of 58 – 19, we’ve talked about making a friendlier number from the 19 by adding one to make 20. I usually have to help them along though. They don’t automatically jump from 19 to 20. Instead, they might say 19 + 10, or some of them might say 19 + 5. At this point, I usually have to prompt them to think about adding one to make a friendly number.

Mrs. Knight: Number talks and number relationships. During the second observation, Mrs. Knight noted that for the next few number talks sessions, students would be exploring the relationship between doubling and halving numbers. This number talk was a continuation of a series of sessions on this topic. Throughout the session, Mrs. Knight used visual models to explore the concept of doubling and halving and challenged students to discover a pattern between the factors of a string of expressions.

Writing 1 x 16 on the board, Mrs. Knight revealed, “I’m going to draw an array to represent the expression 1 x 16.”
Mrs. Knight drew a one by 16 array on the board as demonstrated in Figure 7.

Figure 7. Array model of 1 x 16.

Mrs. Knight asked, “What is the product of 1 x 16?”
Responding as a group, the class stated, “16.”
Mrs. Knight continued, “If I were to double the first factor, what would it become? Ian, do you have an answer?”
Ian stated, “Two.”
Mrs. Knight, after recording Ian’s solution of two on the board, asked, “Okay, and if I were to halve the second factor, what would it become, Jose?”
Jose answered, “Eight!”
Mrs. Knight reiterated, “So, if I double the first factor, the factor changes to two, and if I halve the second factor, it changes to eight. Who can tell me the new expression that was created?”
Ian responded, “2 x 8.”
Mrs. Knight replied, “So we started with an array that showed 1 x 16. After doubling the first factor and halving the second factor, we have a new expression of 2 x 8. What would my array look like now?”
Mrs. Knight allows approximately 10 seconds of wait time for students to process the question.

Mrs. Knight inquired, “Casey, what would the array look like?”

Casey responded, “Two rows and eight columns.”

Mrs. Knight drew a two by eight array on the board as demonstrated in Figure 8.

![Array model of 2 x 8](image)

*Figure 8. Array model of 2 x 8.*

Mrs. Knight questioned, “Does 2 x 8 have the same product as 1 x 16?”

The class, responding as a group, cried, “Yes!”

Mrs. Knight asked, “Okay, how can we prove that, Jackson?”

Jackson answered, “You can just count the boxes in each array. There are 16 boxes altogether in each one.”

Mrs. Knight replied, “So one strategy is to count the number of total boxes in each array. Does anybody have another way to prove that the product of 1 x 16 and 2 x 8 is the same? Callie?”

Callie explained, “Well, 8 + 8 = 16. In the second array, there are two rows of eight, so you can add 8 + 8. In the first array, you can count the first eight boxes and then add eight more. Both arrays have 16.”
Mrs. Knight responded, “Okay, let’s look at another one. What about 4 x 4? Think about the relationship between 2 x 8 and 4 x 4.”

After a wait time of about 20 seconds, Mrs. Knight asked, “Emily, what do you think?”

Emily stated, “The array would have 4 columns and 4 rows.”

Mrs. Knight drew a four by four array as shown in Figure 9.

![Figure 9. Array model of 4 x 4.](image)

Mrs. Knight prompted, “And what is the product of 4 x 4?”

Emily answered, “16.”

Mrs. Knight inquired further, “How did you know that 16 was the product?”

Emily replied, “There’s four in each row, so I added 4 + 4 + 4 + 4.”

Writing the repeated addition sentence on the board, Mrs. Knight asked, “Okay, can anybody see the relationship to what I showed you before? We started with 1 x 16, and then we went to 2 x 8, and now we have 4 x 4. Let’s review. How did we get from the factor of one to the factor of two? Callie?”

Callie answered, “We doubled it.”
Mrs. Knight repeated, “We doubled it. We multiplied times two. And then how did we get from the two to the four? Jackson?”

Jackson stated, “We doubled it.”

Mrs. Knight continued, “So we doubled one to get two, and we doubled two to get four. Now let’s look at the second factors. How did we get from 16 to eight? Audrey?”

Audrey answered, “We subtracted. Wait, we halved it!”

Mrs. Knight pressed further, “We halved it, and when we halve, we divide by. . .”

Audrey, hesitating, responded, “8?”

Mrs. Knight replied, “Not 8. If we divide something in half, how many pieces are there?”

Audrey responded, “4?”

Mrs. Knight, addressing the class, “If we divide a cookie in half, how many pieces would there be?”

As a group, the students responded, “Two!”

Mrs. Knight clarified, “Yes, there would be two pieces! When we halve a number, we are dividing by two. So 16 ÷ 2 = 8, and 8 ÷ 2 equals . . . Emily?”

Emily replied, “Four!”

Mrs. Knight reiterated, “Yes, four!”

Mrs. Knight displayed the relationship between 1 x 16, 2 x 8, and 4 x 4 on the board as shown in Figure 10.
In a final explanation to students, Mrs. Knight stated,

Let’s look at the relationships between the expressions one more time. So with 1 x 16, we doubled the one by multiplying 1 x 2 and got a factor of two. And we halved the 16 and turned it into 8 by doing 16 ÷ 2, which gave us a new expression of 2 x 8. So we multiplied one factor by two and divided the other factor by two. And then we did the same thing to get 4 x 4. We doubled the two and halved the eight. What is the product of 1 x 16, 2 x 8, and 4 x 4?

The students responded, “16.”

Mrs. Knight said, “Yes, we keep getting a product of 16. We’re going to look at more problems with doubling and halving tomorrow and talk about how you can apply this strategy.”

In the debriefing interview, I asked Mrs. Knight about her decision to introduce the doubling and halving strategy to students.

Mrs. Knight said,

Doubling and halving is a strategy that I find very useful, but my students are not very familiar with it. I want them to be able to look for patterns and relationships between numbers, so they have the tools needed to solve problems flexibly.
Number talks is a time for [students] to explore and think about those relationships.

*Figure 11* displays participant themes for number relationships.

*Figure 11.* Participant themes for number relationships.

**Number Talks in Practice**

This study consisted of six subquestions that concentrated on the instructional practices of teachers during number talks. This section provides an analysis of the practices that occurred during the classroom observations. Parrish (2011, 2014) identified five fundamental components for effectively implementing a classroom number talks: (a) a classroom environment that is safe for the sharing of student ideas, (b) a facilitator who questions instead of tells students, (c) mental math strategies that encourage efficient strategies, (d) purposefully selected computation problems that guide students in developing mathematical relationship and patterns, and (e) discourse that is
rich in communicating mathematical knowledge. These five tenets served as the guiding framework when coding the data.

Subquestion A: Classroom Community

The first subquestion focused on how teachers develop classroom community in number talks. Parrish (2011, 2014) emphasized the importance of fostering a respectful learning environment where all students feel safe in sharing their ideas and solutions to problems. I analyzed data from interviews, observations, and fieldnotes using the a priori code of classroom community. The results are reviewed in the following sections.

Mrs. Miller: Classroom community. At the start of this study, I asked Mrs. Miller, “What steps have you taken to develop a classroom community where all kids feel safe during number talks?”

Mrs. Miller reported,

Our learning community extends beyond the scope of number talks. At the beginning of the school year, we create a class constitution and develop our classroom norms. We’re a family, and we have class meetings. We talk about making mistakes, and that mistakes are proof that you are trying, and there’s nothing to be embarrassed about. Overall, they are respectful of each other. They don’t laugh or make fun of each other. One of my major philosophies is maintaining self-esteem, so I never want them to feel like they are not intelligent or anything like that. So I praise, praise, praise, praise, praise a lot. I tell them all the time that we’re a community, and we’re going to support each other.
A review of the observation videos and fieldnotes for this study revealed a strong community of learners in Mrs. Miller’s classroom. Over the course of this study, all students demonstrated respectful behavior during the number talks observations. Students were obviously comfortable in stating and explaining their answers to problems, and they were courteous when agreeing or disagreeing with their peers. Together, they celebrated the efforts and successes of their classmates.

Mrs. Addison: Classroom community. At the beginning of the study, I asked Mrs. Addison to describe the steps she has taken as a classroom teacher to build a sense of community where all students feel safe to share their responses during number talks.

Mrs. Addison replied,

We have class meetings, just in general, about being kind. If I see them not being nice to each other at any point during the day, I’ll stop what we are doing, and we come to the carpet for a class meeting to discuss it. I always tell my students that I want them to be smart, but I also want them to be kind. Kindness to me is more important than being smart. If you’re smart, but you’re mean to people, where is that going to get you in life? So that’s really big for me—being good, moral people as well as being smart. So, we have lots of class meetings to discuss that goal.

During the first number talks observation for this study, the students in Mrs. Addison’s class were respectful to one another. The students supported each other as they talked through various problems. However, a different situation ensued during the second observation. As a student was sharing her strategy for solving a multiplication
problem, she incorrectly stated her strategy for finding the solution to the problem. Instead of allowing the student time to explain her reasoning, a few students in the class responded with laughter. A few other students asked, “What?” or stated, “That’s not right!” As the scenario unfolded, Mrs. Addison asked the class to give the student an opportunity to explain. Mrs. Addison remarked, “Let her finish explaining. And if you have a better way to represent the problem, then you should raise your hand, and I’ll call on you to explain your strategy.” Later during the number talk, a different problem was introduced, and the same student raised her hand to share her solution. Before sharing her strategy, the student asked, “Are they going to laugh at me?”

Mrs. Addison responded to the class, “No, we’re not going to laugh because that’s not being a respectful student, right?” As the student thought about her solution, she indicated that she would like to pass on sharing this time.

During the debriefing interview, I began the conversation by asking Mrs. Addison to share her reflection on the number talks session.

Mrs. Addison responded,

I thought the actual math concept went well, but I was not pleased with my students’ behavior. That was my biggest concern, because they weren’t as kind to each other as they were during the previous observation. I think that made for a lot of students not wanting to participate because they were afraid of getting laughed at or having things said to them. They did better than I expected with the math, but I didn’t like their overall behavior.
Later in the interview, I asked, “After the observation, did you address the students’ behavior that occurred during the number talk?

Mrs. Addison stated,

I addressed both the behavior and the issue of not being nice to each other. I don’t want anyone to feel intimidated when they don’t know an answer. So, I have addressed it, and that it’s inappropriate to make another person feel bad because they don’t know something. We’re all here to learn, and number talks is a time for us to share our thinking and learn from each other.

I further inquired, “Do you think that all students feel safe to share during number talks?”

Mrs. Addison responded,

I thought so until that [observation]. I don’t know what happened—maybe they were just having a bad day. It’s normally not quite as bad. You were here during the first observation, and it was nothing like that, so I’m not sure what was going on that day.

A few days later, I conducted the third observation for this study, and the classroom atmosphere was quite different from what I had observed the previous week. As noted during the first number talks session, the students supported one another as they talked through each problem. It was only near the end of the session where a few students, sharing out of turn, expressed confusion about some of the strategies presented. Some students acknowledged their confusion by calling out, “I’m confused,” while others chose to partake in their own conversations. Observing that students were losing focus,
Mrs. Addison stated, “You’re confused? Well maybe we’ll work on this later. I’ll leave this up here, and we can revisit it, okay?”

Mrs. Knight: Classroom community. During the introductory interview, I asked Mrs. Knight to portray the classroom environment during number talks.

Mrs. Knight remarked,
My students love number talks. It’s a dedicated time where everyone is really paying attention, and it’s a respectful and peaceful time where they are thinking and applying. It’s the one time when they’re not trying to outdo each other. They’re just really thinking about what they’re saying and analyzing what the other person is saying.

I further inquired, “What has been your role in developing that sense of community with your students?”

Mrs. Knight answered,
Honestly, I think it’s something that comes from [my students] being exposed to math talks in prior years. They look forward to it every day. My role is to make sure that we all come together in one place so that we’re focused in one specific area of the room. We honor each other’s thought process, and there is no racing to get to the answer first. Emphasizing the use of hand gestures to communicate during number talks is important so they know this is not a time to shine, but a time to share. But I really think [my students] have been exposed to good math talks prior to this year, so they came to me already aware of the expectations during this time.
A review of the observation videos and fieldnotes for this study indicated a respectful community of learners in Mrs. Knight’s classroom. During each number talk observation, the students exhibited a positive classroom culture where they were willing to construct and share their own knowledge while examining and questioning the reasoning of others.

*Figure 12* displays participant themes for classroom community.

![Diagram](image)

*Figure 12. Participant themes for classroom community.*

Subquestion B: The Role of the Teacher

The second subquestion concentrated on the role of the teacher during classroom number talks. According to Parrish (2011, 2014), the teacher acquires the role of facilitator, questioner, listener, and learner during number talks. Using the a priori code of the teacher’s role, I analyzed data from interviews, observations, and fieldnotes. A review of the findings is discussed in the following sections.
Mrs. Miller: The role of the teacher. During the introductory interview, I asked Mrs. Miller to describe her role during number talks. She responded,

I am the facilitator. It’s my job to make sure that everyone is paying attention and participating, but I really am a facilitator. My role is to make sure the number strings are connected to a specific skill, idea, or concept that I want [my students] to understand. If they are not grasping something that I believe is important, I will say, “Have you considered this strategy?,” and I’ll show it to them. Some kids will adopt it, while others will not. But a lot of times when I do that, like the very next problem I give, half of my kids will try and use the strategy I just presented. So, it really is just being a facilitator. It’s giving them the opportunity, exposing them to concepts, and pointing them in the right direction.

During each classroom observation, Mrs. Miller consistently engaged students in the conversation by asking guiding questions, exploring misconceptions, and modeling problems with manipulatives to develop understanding. The following sections provide evidence for how Mrs. Miller incorporated these components into her classroom number talks.

**Guiding questions.** During each observation, Mrs. Miller continually prompted students to explain and justify their reasoning by asking questions to extend students’ thinking. For example, to check for mathematical understanding, Mrs. Miller asked questions such as “What is the meaning of the denominator?”; “How can we find the number of pieces in the whole?”; “Why is an eighth less than a half?”; “What is an equivalent fraction?”; and “What is the relationship between the three and the six?” Mrs.
Miller also encouraged students to share their reasoning about problems through questions and phrases such as “What do you think?”; Why is the numerator three?”; “Tell me why you agree or disagree”; “What do you notice about the denominators?”; “Explain your thinking”; and “What pattern do you see?” Through intentional questioning, Mrs. Miller promoted purposeful conversations about mathematical problems. Moreover, she was able to observe students’ understanding of the content and identify misconceptions in their thinking.

*Exploring misconceptions.* During the first observation, a student was struggling to correctly name the fraction 2/2. As the situation unfolded, the following conversation ensued.

Mrs. Miller asked, “Olivia, what do you think?”

Olivia responded, “It’s two-tooths.”

Mrs. Miller replied, “Two-tooths? Look at the fraction vocabulary on our anchor chart.”

Olivia, changing her response, says, “Two-twos?”

Mrs. Miller stated, “Think about the meaning of two twos. If I have two twos, how much do I have?”

Olivia replied, “Two twos is four.”

Mrs. Miller continued, “That’s correct. So how do we say this fraction?”

Olivia responded, “One-half?”
Mrs. Miller, writing the fraction 1/2 on the board, said, “So when the fraction has a one in the numerator and a two in the denominator, we say one-half. What do we say when the fraction has a two in the numerator and a two in the denominator?”

Olivia, pausing to think, answers, “Two-halves.”

Rather than simply telling Olivia the correct way to say the fraction, 2/2, Mrs. Miller, instead, guided Olivia in revising her thinking by referencing a classroom anchor chart and well-known fraction to alleviate the misconception.

*Modeling problems with manipulatives.* Throughout the observation period, Mrs. Miller frequently modeled mathematical concepts for students during number talks. For example, number line visuals were used to support students with understanding fractions, and circular and rectangular area models were employed to demonstrate equivalent fraction concepts. During the final observation, Mrs. Miller presented two fractions to the class: 6/8 and 3/8. She then asked, “How would you fill in the blank to this sentence: greater than, less than, or equal to? 6/8 is (blank) 3/8.” After giving a short wait period, the following vignette occurred.

Mrs. Miller asked, “Rachel, what do you think?”

Rachel answered, “6/8 is greater than 3/8.”

Mrs. Miller replied to the class, “Rachel says that 6/8 is greater than 3/8. Does anyone disagree with Rachel?”

Sam stated, “I think that 6/8 is equal to 3/8.”

Mrs. Miller confirmed, “So you say that 6/8 is the same as 3/8? Sarah, you agree with Sam that it’s equal?”
Sarah, nodding in agreement, said, “Yes, they’re equal.”

Mrs. Miller responded, “Okay, I’m curious. Sam, tell me why you think 6/8 is the same as 3/8.”

Sam explained, “Because 3 x 2 equals 6, which is the same.”

Mrs. Miller, identifying the misconception, handed the students some fraction tiles to concretely model the problem. Displaying two red tiles, Mrs. Miller explained, Each red tile represents one whole. I am going to partition each whole into eight equal parts. Sam, I am going to give you 6/8 of this whole. Sarah, I’m going to give you 3/8 of this whole. Look at how many eighths pieces you have in your hand. Sam, how many pieces do you have?

Sam responded, “I have six.”

Mrs. Miller continued, “Sarah, how many eighths pieces do you have?”

Sarah answered, “Three.”

Mrs. Miller clarified, “Sam, who has more pieces, you or Sarah?”

Sam replied, “Me.”

Mrs. Miller questioned, “Alright, so is 6/8 the same or equivalent to 3/8, Sam?”

Sam responded, “No.”

Mrs. Miller further inquired, “Why not?”

Sam explained, “Because there’s more 6/8 than 3/8.”

By using concrete objects to model the problem, Mrs. Miller aided her students in making sense of the problem.
Mrs. Addison: The role of the teacher. During the introductory interview, I asked Mrs. Addison, “As the teacher, what do you think your role is during classroom number talks?”

Mrs. Addison declared,

My role during number talks is to facilitate. It is my responsibility to write [the students’] answers on the board. I monitor the wait time. I call on [students], and I ask them to share. Then, I ask their classmates if they agree or disagree and why. It is very difficult for me not to interject and tell them how to solve the problem, but I’m working on that.

After watching the video from the first number talks observation, Mrs. Addison noted some personal changes to her facilitation practices. For example, Mrs. Addison presented the following problem to students: “Find the perimeter of the shape. If the area is 30 square feet, what is the perimeter?” Figure 13 shows the visual representation that Mrs. Addison displayed to illustrate the problem.

![Figure 13. Visual representation of perimeter and area problem.](image-url)
After allowing a 20-second wait period, Mrs. Addison asked Miguel to explain his thinking to the problem. Miguel said, “I think that the other side has to be six feet.”

Mrs. Addison questioned, “Which other side? Can you come show me?” Miguel walked to the board and labeled the opposite side as six feet as shown in Figure 14.

Figure 14. Miguel’s strategy for finding the perimeter of the rectangle.

Miguel then explained, “$6 + 6 = 12$, and $30 – 12 = 18$. Since there are two sides, I divided. $18 \div 2 = 9$, so each side equals nine.”

Mrs. Addison, addressing the class, said, “Okay, show me if you agree with Miguel’s answer.” All students used the hand signal for agreement to indicate they agreed with Miguel’s answer. Mrs. Addison responded, “Alright, I need you to turn and talk with a partner one more time because this answer is not the correct answer. Look at the problem again and see what could be wrong with it.”

After the students had an opportunity to discuss the problem, Mrs. Addison said, “I think I hear some revelations. Ansley, would you like to share your thinking?”

Ansley explained, “It says 30 square feet for the area. It only says square feet when we are finding the area.”
Ben, joining in the conversation, said, “Oh, I figured it out, but I was confused because the other side is longer than that side. I didn’t think it could be five feet.”

Mrs. Addison said, “That’s my drawing mistake, so don’t pay attention to the length of the sides in the drawing.”

Ben continued, “I did 6 \times 5 because we are finding the area, which is 30 square feet. So the other side is 5 feet.”

Mrs. Addison asked, “Can you come show us where the five feet goes?”

Ben labeled the dimensions of the rectangle as shown in Figure 15.

![Figure 15](image)

Figure 15. Ben’s strategy for finding the perimeter of the rectangle.

Mrs. Addison then asked Ansley, “What is the perimeter of the rectangle?”

Ansley said, “Well, 5 + 6 = 11, and then you add 5 + 6 again, and you get 11. And 11 + 11 = 22, so the perimeter is 22 feet.”

In the debriefing interview following this observation, Mrs. Addison noted, “There are little things that I need to fix so [my students] don’t have misconceptions that are unnecessary. On my part, I need to make sure that my models are drawn to scale.”

Furthermore, Mrs. Addison noted,
There were just little things that I noticed about myself when watching the video, such as the spacing on the board when recording strategies. I was writing all over the place, and even I was struggling to follow along. It was too jumbled, so I need to make sure that I’m a little neater so [my students] don’t get confused. I need to write clearer so those who don’t understand are able to follow the conversation.

Mrs. Knight: The role of the teacher. Prior to the first observation, I asked Mrs. Knight, “What is your role during classroom number talks?”

Mrs. Knight answered,

My role is definitely that of a facilitator, but my students are very quiet so I find my voice in the conversation more than I would like. I really want to get to the point where I barely hear my voice, and I’m trying to figure out how to balance that during number talks. But I definitely consider myself a facilitator in that setting. My students take number talks very seriously, and while they are willing to share, I feel like I direct the conversation more than I should. Because so many of them have ideas to share, I find that most of the time we only do about two problems, and that has been a struggle for me.

Two themes emerged from the data analysis of Mrs. Knight’s facilitation of number talks: wait time and recording.

Wait time. A review of the observation transcripts revealed that Mrs. Knight presented only one problem during the first observation: 6 x 15. During the 22-minute observation, the students shared a total of five strategies for this expression. During the
debriefing interview, I asked Mrs. Knight to share her reflections on the session. Mrs. Knight replied,

I actually got a lot of insight into some of my students’ thinking, so that was helpful. I liked the pace, but sometimes I feel like I’m not getting enough out of a session just based on the limited amount of time that is dedicated to number talks. The wait time was excruciating for me! That is where I struggle—how much wait time is appropriate without taking away the think-time needed to solve the problem?

During the remaining two observations, Mrs. Knight reduced the amount of wait time for each problem, and as a result, she was able to present more problems to the class.

Recording. Another conversation that arose during the first debriefing interview was the importance of accurate recording. This exchange surfaced due to a recording error during the first observation. The following vignette illustrates the scenario that took place during the number talk.

After displaying the problem 6 x 15 on the board, Mrs. Knight said, “Jackson, can you share your strategy for solving?”

Jackson explained, “I decomposed the 15 into 10 + 5.”

Mrs. Knight recorded Jackson’s strategy on the board, and said, “What did you do next?”

Jackson responded, “I multiplied 10 x 6, which equals 60. Then, I did 5 x 6, and I skip counted six, five times.”
Mrs. Knight clarified, “Okay, so you skip counted six, five times. Tell me what that would look like if I write it down.”

Jackson stated, “$6 + 6 + 6 + 6 + 6$, and that equals 30.”

Mrs. Knight asked, “Okay, how did you get that to be 30? What did you do?”

Jackson answered, “I added six to another 6.”

Mrs. Knight questioned, “And what did you get?”

Jackson declared, “12.”

Mrs. Knight, recording Jackson’s addition calculations as demonstrated in Figure 16, said, “Okay, then what?”

Jackson continued, “Plus six equals 18, plus six equals 24, plus six equals 30, and I know that $30 + 60 = 90$.

![Figure 16. Mrs. Knight’s recording of Jackson’s strategy.](image)

In the debriefing interview that followed, I asked Mrs. Knight to review the recording of Jackson’s strategy. I said, “I noticed that when you recorded $18 + 6$ and 24
+ 6, you connected them with an equals sign, suggesting that 18 + 6 is the same as 24 + 6, which is the same as 30.

Mrs. Knight responded, “Yes, I see what you’re saying. Since 18 + 6 does not equal 24 + 6 or 30, they should not be connected with an equals sign. That makes sense.” As the study progressed, Mrs. Knight was more vigilant in her recording practices, and I observed no additional errors.

*Figure 17* displays participant themes for the role of the teacher.

*Figure 17. Participant themes for the role of the teacher.*

Subquestion C: Mental Computation Strategies

The third subquestion investigated how mental computation strategies support students with mathematical understanding as observed by teachers. Researchers agree that an emphasis on mental computation is necessary for developing number sense in students (NCTM, 2000; NRC, 2001; Parrish, 2011, 2014; Reys, 1985; Reys, 1984).
Parrish (2011, 2014) identified mental math computation as a key tenet of number talks because of its role in developing efficient strategies. Throughout the research study, I conducted an analysis of teacher perceptions and actions in supporting students’ mental math computation strategies through number talks. This section provides an overview of the results of the study.

Mrs. Miller: Mental computation strategies. During the initial interview, I asked each teacher participant to share her perceptions on the role of classroom number talks and the influence of the practice on students’ mental computation strategies. I asked, “Based on your observations, how has number talks impacted students’ mental computation strategies?” In her response, Mrs. Miller offered mixed reviews on the impact of number talks on mental computation.

Mrs. Miller stated,

Because of number talks, [my students] will calculate more freely in their heads. As a matter of fact, sometimes I want them to use paper and pencil to solve problems, but instead, they do the math in their heads, and they make mistakes. I don’t think number talks have impacted [my students’] mental math abilities as much as just their reasoning. I feel like it’s impacted their reasoning more than their mental math because a lot of my students still rely on their fingers when calculating. They don’t always apply the mental strategies we’ve discussed. I think [number talks] encourage mental math, but I don’t always see it carrying over to other situations.
During the three number talks observations, Mrs. Miller primarily focused on developing number sense and number relationships with fraction concepts. The three sessions concentrated on identifying fractions on a number line, fraction equivalency, and fraction inequalities. Solving computation problems with mental strategies was not included in the three observations for this study.

Mrs. Addison: Mental computation strategies. During the initial interview, I asked Mrs. Addison to share her perceptions on the role of number talks and their impact in developing mental computation strategies in third-grade students. I inquired, “How have number talks influenced students’ mental computation strategies?”

Mrs. Addison remarked,

A few [students] are able to use mental strategies and figure out problems in their head. Sometimes I see students with their head and eyes towards the ceiling trying to solve [a problem] in their head. I do find a lot of them using mental math. On their assignments, sometimes they’ll just put the answer, and it will be the correct answer, and I have to ask them to explain how they found it. So I think that doing number talks helps to develop that skill.

In one of the observations for this study, Mrs. Addison opted to focus on strategies to solve multiplication problems within 100. Mrs. Addison displayed the problem of 16 x 4 on the board for students to solve. Some of the students were hesitant to try the problem, exclaiming “I’m not good at solving in my head,” or “I need a piece of paper.” After some encouragement, Mrs. Addison proceeded to give a wait time of approximately two minutes before asking students to share their strategies for mentally
solving the problem. During the wait-time period, Mrs. Addison recorded the names of students on the board who indicated they had a solution to the problem.

At the end of the wait period, Mrs. Addison asked, “Haley, how did you solve for the product of 16 x 4?”

Haley responded, “I broke down 16.”

Mrs. Addison said, “Okay, to what?”

Haley replied, “To 10 and 6.”

Recording the strategy on the board, Mrs. Addison, reiterated, “10 and 6. Alright, then what?”

Haley continued, “Then I multiplied 6 x 4 = 24, and 10 x 4 = 40. Then I added.”

Mrs. Addison recorded Haley’s strategy and then asked, “You added what?”

Haley explained, “I added 24 and 40 to get 64” (see Figure 18).

![Figure 18](image-url)  
*Figure 18. Haley’s strategy for calculating 16 x 4.*

Mrs. Addison then asked the class, “Does anybody have a different strategy for solving 16 x 4, Ansley?”

Ansley stated, “I added 16 four times.”
Mrs. Addison confirmed, “You added 16 four times?”

Ansley maintained, “Yes, and so I added 16 + 16 which is 32, and then I knew the other pair of 16s would be 32, so I doubled 32 and got 64.”

Mrs. Addison, recording Ansley’s strategy on the board, explained, “I like how Ansley used doubles to find the answer.” *Figure 19* displays Ansley’s strategy.

![Figure 19. Ansley’s strategy for calculating 16 x 4.](image)

In the final debrief for this study, Mrs. Addison noted that prior to this study, computation problems and the sharing of mental strategies were typically the focus of her number talks sessions. She acknowledged that before this study began, students were not accustomed to the practices of student-to-student discourse, estimation, and error analysis problems during number talks. Mrs. Addison stated, “I always thought [number talks] were a set regimen, but after our conversations, I’m starting to expand the types of problems I introduce to students during that time.”

Mrs. Knight: Mental computation strategies. At the start of the study, I asked Mrs. Knight to share her perceptions on the role of number talks and their influence on
students’ mental computation strategies. I asked, “How has number talks impacted students’ mental computation strategies in your classroom?”

Mrs. Knight explained,

Several of my students try to use mental math strategies on their assessments, and it’s great that they are relying on those strategies. But I also need to see the reasoning for their answers, and they don’t always include that. I try to explain that I need evidence of their thinking. In number talks, that’s a verbal conversation, but on assignments, I need to see the written language to understand their reasoning. Sometimes I’m surprised by how quickly they are able to solve problems because of mental math. And I’ve noticed that the more consistent I am with implementing number talks, the more [my students] rely on mental computation.

During the first number talks observation, Mrs. Knight concentrated on multiplying numbers within 100 using mental computation strategies. Mrs. Knight began the session by displaying the expression 6 x 15 on the board for students. After introducing the problem, she allotted a wait time of almost two minutes for students to form a strategy and solution for the problem. At the end of the wait period, Mrs. Knight asked students to share their solutions for the problem. One student shared that his answer to the problem was 90, and Mrs. Knight recorded the student’s solution on the board. All other students signaled they were in agreement with the solution of 90. As a result, Mrs. Knight sought student strategies for mentally finding the solution of 90. Mrs.
Knight questioned, “Alright, who has a strategy for coming up with 90? Leah, can you explain your strategy for finding the product of 90?”

As Mrs. Knight recorded the strategy on the board, Leah explained, “I broke apart 15 into 5 and 10, and then I multiplied 6 x 5, and that equals 30. Then, I multiplied 6 x 10 and got 60. And that equals 90 because 30 + 60 = 90.”

Mrs. Knight recorded Leah’s strategy on the board as shown in Figure 20.

```
6 x 15
/\  \\
5 + 10

6 x 5 = 30
6 x 10 = 60
30 + 60 = 90
```

*Figure 20.* Leah’s strategy for calculating 6 x 15.

Mrs. Knight then asked, “Alright, does anybody else have a different strategy? What was your strategy, Jose?”

Jose, validating Leah’s strategy, explained to the class, “I did something similar to Leah’s strategy, but after I broke up the 15 into 5 and 10, I then broke up the 10 into 5 + 5.”

Mrs. Knight clarified, “So you went further and broke up the 10?”

Jose, nodding in agreement, responded, “Yes, into 5 + 5.” And then I did 6 x 5 which equals 30 three times, so 6 x 5 = 30, 6 x 5 = 30, and 6 x 5 = 30, and 3 x 30 = 90.”

Mrs. Knight recorded Jose’s strategy on the board as demonstrated in Figure 21.
Afterwards, Mrs. Knight announced to the class, “Okay, we have two different approaches to multiplying 6 x 15. I want you to turn and talk about Leah’s strategy and Jose’s strategy. How are the two strategies alike, and how are they different?”

Prior to the first observation, Mrs. Knight mentioned that the turn and talk was not an approach that she had formerly tried during number talks. During our initial interview, I inquired, “Do you ever have students turn and talk to discuss problems during number talks?”

Mrs. Knight replied, “That is one of my weaknesses. I’ve observed the practice of turn and talks through a colleague, but I have not learned to appreciate the usefulness of it, so I definitely underutilize it in practice.”

After observing the implementation of the turn and talk during the first observation, I revisited the notion during the debriefing interview and asked Mrs. Knight to reflect on the process. I questioned, “Was that the first time that the students have participated in a turn and talk?”

Mrs. Knight answered,
Yes, this is the first time I’ve implemented the turn and talk component, and I was really surprised at the language and the discourse between the students. I heard some thinking of students who don’t necessarily speak out in the whole group, but they were willing to share their thoughts and reasoning on a strategy with a peer.

At the conclusion of the first number talks observation, Mrs. Knight requested that students return to their seats, and as a conclusion to the conversation, she invited students to develop a written reflection on Leah’s strategy and Jose’s strategy in their math journals. In the debriefing interview, I asked Mrs. Knight to expand on her reasoning for this practice.

Mrs. Knight shared,

The closing is something that I’m trying to bridge in more. I want [my students] to illustrate and write about mathematics more often. And we’re still trying to get there because I didn’t get a lot of product and response out of that exercise. I maybe had five kids who were able to model and explain their thinking, so I’m trying to make sure that we’re consistently bridging the language of verbal reasoning to written reasoning.

*Figure 22* displays participant themes for mental computation strategies.
Figure 22. Participant themes for mental computation strategies.

Subquestion D: Purposeful Computation Problems

The fourth subquestion explored how teachers select purposeful computation problems for number talks. Parrish (2011, 2014) conveyed that classroom number talks involve closely related computation problems, called number strings, which consist of a series of computation problems purposefully constructed to demonstrate mathematical patterns and relationships between numbers. The a priori code of purposeful computation problems was used to analyze data from interviews, observations, and fieldnotes. An analysis of the data is discussed in the ensuing sections.

Mrs. Miller: Purposeful computation problems. At the start of the study, I asked Mrs. Miller to describe her process for selecting number strings for her classroom number talks. Throughout the study, Mrs. Miller acknowledged that she used number talks as a preview to upcoming content in her regular mathematics class.

Mrs. Miller stated,
Compared to previous years, this group of students has grasped concepts more quickly because of the preview they’re getting through number talks. The number talks are giving them the preknowledge needed to later apply to the problems we do in math class.

A review of the number talks observations over the course of the study period indicated that Mrs. Miller consistently selected a series of purposeful problems with the intent of developing students’ knowledge for approaching fraction content. Table 6 presents an overview of the number strings presented by Mrs. Miller during each number talk observation.

Table 6

*Mrs. Miller’s Number String Overview*

<table>
<thead>
<tr>
<th>Observation Number</th>
<th>Content Focus</th>
<th>Number of Problems</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Fractions on a number line</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>Fraction equivalence and inequalities</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>Fraction inequalities</td>
<td>6</td>
</tr>
</tbody>
</table>

Furthermore, Mrs. Miller noted that she frequently provided separate number talks for students who require remediation based on formative assessment data. Mrs. Miller stated,

I have a list of students who need additional support with the make-a-ten addition strategy. I have three students who consistently say, for example, that $6 + 4$ is $11$. 
Because of that, I’m going to facilitate a small-group number talk where we can focus on basic addition facts.

Mrs. Addison: Purposeful computation problems. During the initial interview, I asked Mrs. Addison the following question, “Prior to each number talks session, how do you determine what number strings best meet the needs of your students?”

Mrs. Addison responded,

It really depends from day-to-day. Sometimes I try to line them up with the content that we are currently doing in class, or content that we have previously covered but need to revisit. At times, I use my assessment data to determine the types of problems that were difficult for students, and then I incorporate similar problems in my number talks so we can review and discuss them.

Throughout the study, Mrs. Addison expressed interest in trying different types of problems during her number talks. As a result of our interview conversations, Mrs. Addison introduced a true-false problem scenario to encourage student-to-student discourse and reasoning, an error-analysis problem to examine the importance of place value, and estimation to determine the reasonableness of answers. Because of Mrs. Addison’s enthusiasm to try different problem types, the content focus was not obviously sequential between observations. However, the number strings within each session appeared well-designed. Typically, the conversation began with simpler problems, which eventually led to more thought-provoking problems by the end of the session. Table 7 presents an overview of the number strings presented by Mrs. Addison during each number talk observation.
Mrs. Knight: Purposeful computation problems. Prior to the first observation, I asked Mrs. Knight to describe her process for determining the number strings that best meet the needs of her students. Mrs. Knight replied,

It really depends. Sometimes my number strings are based on the content that we are discussing in class because I want to understand how they are reasoning about current concepts. Sometimes, I use the number talks book to select strings that develop foundational skills with computation. Sometimes, I see that my students are struggling with something, and I use number talks as an opportunity to review. So, I jump back and forth depending on the needs of my students.

During the initial interview, Mrs. Knight acknowledged her difficulties with maintaining time during number talks. Because of this, Mrs. Knight introduced only one problem during the first observation: 6 x 15. Although multiple strategies for solving the problem were presented, students did not have an opportunity to see the relationship to other problems. In the debriefing interview, I asked, “If you had continued the number string, what would have been your next problem?”

Table 7

Mrs. Addison’s Number String Overview

<table>
<thead>
<tr>
<th>Observation Number</th>
<th>Content Focus</th>
<th>Number of Problems</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Area and perimeter</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>Multiplication with the distributive property</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>Estimation and error analysis</td>
<td>3</td>
</tr>
</tbody>
</table>
Mrs. Knight replied in a confident tone, “I would have tried $6 \times 30$ because of the doubling component that I really haven’t used as much as I want to, so I was hoping that we could explore that concept.”

In the observations that followed, Mrs. Knight continued to incorporate number strings that highlighted doubling and halving. During the second and third observations, Mrs. Knight introduced several problems to demonstrate the concept. An analysis of the data revealed that problems introduced during the second observation were more practice-based to allow students an opportunity to understand the meaning of halving numbers. For example, Mrs. Knight asked, “What number is half of 12?” or “What number is half of six?” Because of this, an authentic number string was not evident.

During the third observation, Mrs. Knight revisited the concept of doubling and halving, but this time, there was an obvious relationship between the problems in the number string. Mrs. Knight presented the following string to investigate the concept: $1 \times 16$, $2 \times 8$, and $4 \times 4$. After exploring the concept with visual models (see number relationships for examples), students reasoned that in a multiplication equation, when one factor is doubled and the other factor is halved, the product remains the same. Table 8 depicts an overview of the number strings presented during each number talk observation.
Table 8

*Mrs. Knight’s Number String Overview*

<table>
<thead>
<tr>
<th>Observation Number</th>
<th>Content Focus</th>
<th>Number of Problems</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Multiplication within 100</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>Halving</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>Doubling and halving</td>
<td>3</td>
</tr>
</tbody>
</table>

*Figure 23* displays participant themes for purposeful computation problems.

*Figure 23*. Participant themes for purposeful computation problems.

Subquestion E: Student Discourse

The fifth subquestion for this study concentrated on the role of classroom number talks in promoting student discourse as reported by teachers. Parrish (2014) asserted that the consistent implementation of number talks fosters the development of computational fluency and promotes sense making in mathematics by involving students in classroom discourse that leads to accurate, efficient, and flexible computation strategies. Moreover,
researchers widely acknowledge that discourse is essential to developing mathematical reasoning and understanding (Franke et al., 2009; Lampert, 1990; NCTM, 2000). Using the a priori code of classroom discussions, I analyzed data collected from interviews, observations, and fieldnotes. The following sections present an analysis of the data.

Mrs. Miller: Classroom discussions. During the initial interview, I asked Mrs. Miller, “How do you promote mathematical discourse during number talks?”

Mrs. Miller replied,

Discourse is a natural part of number talks. Most of the time I call on a student who has indicated they have a solution or strategy for solving the problem. If I find that the same students are sharing, I’ll say something like “I want someone to raise their hand who has not spoken yet today.” There are some students who I know are not comfortable with certain skills, and so I usually pull them for a small-group number talk at a different time.

I further inquired, “Do you incorporate any turn and talk during number talks to promote student-to-student discourse?”

Mrs. Miller stated,

No, my students do not talk unless I call on them. The turn and talk usually comes during the mini-lesson for math. For number talks, we try and keep it quick, so they are supposed to stay quiet and not have any side talk.

Throughout the course of the study, the only type of discourse observed was student-to-teacher discourse. The presence of student-to-student discourse was not evident.
Mrs. Addison: Classroom discussions. Prior to the first observation, I asked Mrs. Addison, “How do you promote mathematical discourse during number talks?”

Mrs. Addison responded,

That’s a struggle for me. I’m still working on developing community and kindness towards one another. Sometimes, a student will share an answer that is not correct, and someone will say, “That’s not the answer!” I always tell them that we’re going to let everyone have an opportunity to share, and then if they have something constructive to say or discuss, they’ll have their chance. Sometimes I’ll see a couple of [students] talking to each other and saying, “Oh, this is what you need to do,” but I don’t see that a lot. So I’m still working on that.

I further inquired, “Have you tried incorporating a turn and talk?”

Mrs. Addison remarked, “No, I have not tried that, although I am interested in it.”

Throughout the study, Mrs. Addison began to incorporate student turn and talk as a regular practice during number talks. An overview of two of the turn-and-talk vignettes in Mrs. Addison’s classroom are included earlier in this chapter as part of the guiding research question on number sense and number relationships. The student turn and talks became a regular point of interest during the debriefing interviews, as Mrs. Addison noticed their impact on student learning. As referenced earlier in the chapter, Mrs. Addison stated,

I never incorporated the turn and talks prior to this study, and I think it’s a good approach for getting everyone engaged in the conversation. I see more of [the
students] getting involved and talking instead of being afraid to share their answer. With the turn and talk, they have a chance to reason through their answer and receive peer feedback before sharing it with the entire class.

An analysis of the data revealed that student-to-teacher discourse was the primary form of discourse prior to the start of this study, but student-to-student discourse quickly became an integral part of number talks as the study progressed.

Mrs. Knight: Classroom discussions. During the introductory interview, I asked Mrs. Knight, “How do you promote student discourse during number talks?” Mrs. Knight replied,

Well, I’ve tried a number of things. I do have students who will hide, and sometimes I tell them, “I’m going to wait until everyone shows me that they have an answer.” Then, they realize they are going to have to come up with a response, and I may call on one of them. And I know they really don’t want to participate, but it’s my way of letting them know that if they participate, they’ll get more comfortable giving answers. So, that’s one of my strategies. At other times I say, “I’m going to wait for everyone because I know you all are mathematicians. You have ideas in your head, and I know you can solve this,” and usually my students who are more hesitant will eventually put their hands up to share. I also say, “Is there a different strategy?”, and that tends to get more of the student’s talking because sometimes they have a strategy they don’t want to say because they know it’s not necessarily the most efficient. But, if I’ve written down maybe two or three strategies, and I ask for a fourth, I will get those students who have other,
less efficient strategies, to share. So just letting them know they have the opportunity tends to get the discourse started, but for the students who are the most hesitant, I have to kind of drag it out a little bit to get them involved.

I then asked, “Do you ever have students turn and talk to discuss problems during number talks?”

As referenced earlier in the chapter as part of the guiding research question on mental computation strategies, Mrs. Knight stated, “That is one of my weaknesses. I’ve observed the practice of turn and talks through a colleague, but I have not learned to appreciate the usefulness of it, so I definitely underutilize it in practice.”

During the first observation, Mrs. Knight incorporated a student turn and talk to reflect on a student’s strategy for solving 6 x 15. An overview of the turn-and-talk vignette in Mrs. Knight’s classroom is located earlier in this chapter as part of the guiding research question on mental computation. During the debriefing interview, Mrs. Knight was complimentary of the student interaction, stating that she “heard some thinking of students who don’t necessarily speak out in the whole group, but [who] were willing to share their thoughts and reasoning on a strategy with a peer.” This was the only time that I observed student-to-student discourse in Mrs. Knight’s classroom during the study. The data revealed that the main form of number talks discourse in Mrs. Knight’s classroom was student-to-teacher discourse.

*Figure 24* displays participant themes for classroom discussions.
Subquestion F: Teacher Questioning

The sixth subquestion for this study focused on the types of questions that teachers ask during number talks to elicit student responses. Franke et al. (2009) found that teacher questioning is critical to student thinking and reasoning in mathematics. Using the a priori code of questioning, I analyzed data from interviews, observations, and fieldnotes. The following sections provide a discussion of the review of the findings.

Mrs. Miller: Questioning. Over the course of the three observations, Mrs. Miller continuously elicited responses from her students through questioning. Mrs. Miller never made assumptions about her students’ reasoning. Instead, she questioned and prompted students to seek understanding of their thought processes. The following vignette is an example of the types of questions that Mrs. Miller typically asked during number talks.

Mrs. Miller, writing two fractions on the board, said, “Okay, let’s try this one. 1/2 and 1/3. Is 1/2 greater than, less than, or equal to 1/3? Sarah, what do you think?”
Sarah answered, “I think that 1/2 is less than 1/3.”

Mrs. Miller, recording the less than symbol on the board, inquired, “Why do you think that 1/2 is less than 1/3?”

Sarah replied, “Because 1/2 has less pieces than 1/3 would have. Two is less than three.”

Mrs. Miller continued, “Okay, so in 1/2, the whole has less pieces than in 1/3. Amanda, you are disagreeing. What do you think it is?”

Amanda stated, “I think 1/2 is bigger than 1/3.”

Mrs. Miller prompted, “Let’s say the proper vocabulary.”

Amanda restated, “1/2 is greater than 1/3.”

Mrs. Miller, recording the greater than symbol on the board asked, “Okay, and why do you think that, Amanda?”

Amanda explained, “I think that 1/2 is greater because you can make bigger pieces with a half than with a third.”

Holding fraction tiles to model the problem, Mrs. Miller responded, “I am holding fraction tiles to represent 1/2 and 1/3. I am representing 1/2 in this hand, and 1/3 in this hand. How many pieces do I have in each hand?”

Olivia answered, “You have one piece that is 1/2 of the whole in that hand, and another piece that is 1/3 of the whole in that hand.”

Mrs. Miller confirmed, “Right! I have one piece in each hand. Why? Russell, can you explain?”
Russell said, “Because one is the numerator, and it tells how many parts of the whole we are talking about.”

Mrs. Miller stated, “Exactly! Our numerator tells us how many. It’s our counting number. Now Sarah, look at the pieces I’m holding in my hands. Which one is greater?”

Sarah replied, “1/2.”

Mrs. Miller responded, “1/2 is greater than 1/3. Why?”

Sarah continued, “Because if you have a smaller number, that means the pieces are larger.”

Mrs. Miller pressed further, “Very good! But why? Tell me why a smaller denominator means a larger piece?”

Sarah explained, “Because you split it into lesser pieces.”

Mrs. Miller confirmed, “Exactly! If you’re going to share brownies with some friends, and you really like brownies, would you rather share it with three friends or 10 friends?”

Sarah responded, “Three friends, because that means I’d get a larger piece than if I shared it with 10 friends.”

Mrs. Miller replied, “Yes! Good job!”

Throughout the study, Mrs. Miller consistently asked “why” after each response. If students were unable to explain, she modeled with visuals and manipulatives to support their thinking. Additionally, as demonstrated in the vignette, she always encouraged students who provided inaccurate answers by providing an opportunity to revise and
explain their thinking to ensure they had grasped the understanding needed to be successful with the concept.

Mrs. Addison: Questioning. During the initial interview, Mrs. Addison revealed that she had set a personal goal of talking less during instruction in order to encourage student autonomy in thinking and figuring out problems for themselves. Mrs. Addison noted, “It is very difficult for me not to interject and tell them how to solve the problem, but I’m working on that.” Throughout the study, many of the questions that Mrs. Addison asked were directly related to computation and more clarifying in nature. For example, during the second observation, Mrs. Addison presented the problem 18 x 5 to students. Previously, students had solved 18 x 4. While presenting the problem, Mrs. Addison said, “I want to see if you recognize anything from the previous problem when you solve this one.”

After a wait time of approximately 40 seconds, Mrs. Addison said, “Okay, 18 x 5. Ava, what’s your strategy?”

Ava answered, “I added five, 18 times.”

Mrs. Addison clarified, “You did five, 18 times? Did you add them up as you went along?”

Ava responded, “Yes.”

Mrs. Addison continued, “Okay, what did you get for your answer?”

Ava answered, “90.”

Mrs. Addison asked, “You got 90?”

Ava confirmed, “Yes.”
Mrs. Addison continued, “Did anybody do a different strategy? Kylie, what was your strategy?”

Kylie explained, “Well, I know from the last problem that 18 x 4 is 72, so then I just added 18.”

Mrs. Addison replied, “You just added 18? And what did you get?”

Kylie answered, “90.”

Mrs. Addison confirmed, “90? Okay, one more strategy! Ben, what was your strategy?”

Ben said, “I decomposed the 18 into 10 and 8.”

Mrs. Addison replied, “10 and 8? Okay, then what?”

Ben replied, “And then I did 10 x 5 and got 50.

Recording the strategy on the board, Mrs. Addison stated, “Okay.”

Ben continued, “And then 8 x 5 equals 30.”

Mrs. Addison asked, “Equals how much?”

Ben replied, “I mean 40. And 50 + 40 = 90.”

Mrs. Addison asked, “Alright, do we agree or disagree?” The students indicated with a hand signal that they agreed with the solutions and strategies.

Data revealed that, in a few instances, Mrs. Addison posed questions that promoted precision, reasoning, and understanding. For example, during the first observation, a student was explaining how to find the perimeter of rectangle. After obtaining an answer of 24 for the perimeter, Mrs. Addison prompted, “The perimeter equals 24 what?”
The student responded, “24 inches.”

Mrs. Addison asked, “And what about the area?”

The student replied, “It would be 36 square inches.”

Later during the observation, another student was describing how to find the side lengths of a rectangle with a perimeter of 18 feet. A width of four feet was given for one of the side lengths.

The student explained, “If the width of that side is four feet, then the width of the other side is four feet. That means that the length is five feet, so you need to label the other two sides five feet.”

Mrs. Addison inquired, “Okay, do you want to explain why you did that?”

The student replied, “Well, if you add 4 + 4, that’s 8. And 5 + 5 = 10, so that’s going to equal 18.”

Mrs. Addison inquired further, “It’s going to equal 18? How did you get 18?”

The student responded, “I added.”

Mrs. Addison said, “You added. Does anyone have another solution for how they solved it?”

In another observation, Mrs. Addison provided opportunities for students to consider the reasonableness of their answers. In this scenario, students were considering the problem 18 x 6. Mrs. Addison listed four possible solutions on the board: 24, 96, 108, and 116. Mrs. Addison then asked, “Which answer can be eliminated right away?”

A student responded, “24.”

Mrs. Addison inquired, “Why can we eliminate 24?”
The student continued, “Because you’re multiplying 18 x 6.”

Mrs. Addison prompted further, “Okay, so we’re multiplying 18 x 6, and . . . ?”

The student explained, “And if you add 18 + 18, that would be 36, so you can eliminate 24 because 36 is greater than 24.”

Mrs. Addison restated, “So she said that if you added 18 + 18, you have 36, which is greater than 24. Does anybody have a different reason for why you can eliminate 24?”

Throughout the conversation, Mrs. Addison continued to press students to explain their reasoning of the problem.

Mrs. Knight: Questioning. During the initial interview, Mrs. Knight remarked that several of her students qualified for ESOL services, and because they lacked confidence with the language, her students were often very quiet during number talks, which sometimes resulted in her voice dominating the conversation.

I inquired, “How do you typically respond when a student is struggling to explain the strategy used to solve a problem?”

Mrs. Knight declared,

I pose questions to support their thinking. I don’t let them off the hook, but I will allow them to ask someone to help them. Many of them are just too shy to even express they need help, so they just sit there. I’ve tried to let them know that they can ask someone else for help. And they don’t utilize it that much, but they also know that I won’t except nothing. Because they know the expectation, a lot of times they’ll just say something to get through it, and sometimes I have to provide words for them to build on.
An analysis of the data showed that Mrs. Knight used questioning as a method for encouraging student responses during number talks. Because many of the students were hesitant with the language, they were reluctant to share aloud with the class. Through questioning, repetition, and clarification, she was able to boost students’ confidence to share their thinking with the class, as demonstrated in the following scenario.

After posing the problem 6 x 15 and recording student strategies for solving the problem, Mrs. Knight inquired, “Does anybody have any other strategies to share? Hector, what’s your strategy?”

Hector responded, “Mine is kind of like Jackson’s with addition, but I added those two sixes and got 12, and then the other two sixes is 12, so I . . .”

Mrs. Knight interrupted, “Okay, slow down for me because I think I’m confused. You added two sixes and got 12?”

Hector confirmed, “Yeah, and then . . .”

Referencing Jackson’s strategy, Mrs. Knight asked, “In this final 30? Come on up and tell me more so I can understand. Start from the beginning.”

Hector, standing at the board, explained, “So it’s kind of like Jackson’s with addition, but I added these two sixes to get 12, and then these two sixes to get 12.”

Hector continued, “And 10 + 10 = 20, so then I added two with the other two and found four. And 20 + 4 = 24. I knew that 6 + 4 = 10, so 24 + 6 = 30.”

Mrs. Knight recorded Hector’s strategy as shown in Figure 25. She remarked, “So basically, you were just breaking down his repeated addition, and you used different strategies to add your sixes.”
Hector confirmed, “Yes.”

<table>
<thead>
<tr>
<th>Jackson’s Strategy</th>
<th>Hector’s Strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$6 \times 15$</td>
<td>$6 \times 15$</td>
</tr>
<tr>
<td>$\begin{array}{c} 10 + 5 \ 10 \times 6 = 60 \ \underline{12 + 6} \ 18 + 6 = 24 + 6 = 30 \ 6 + 6 + 6 + 6 + 6 = 30 \ 30 + 60 = 90 \end{array}$</td>
<td>$\begin{array}{c} 10 + 5 \ 10 \times 6 = 60 \ 6 + 6 = 12 \ 6 + 6 = 12 \ 10 + 10 = 20 \ 2 + 2 = 4 \ 24 + 6 = 30 \ 30 + 60 = 90 \end{array}$</td>
</tr>
</tbody>
</table>

*Figure 25. Hector’s strategy for calculating $6 \times 15$.*

During the debriefing interview, I asked Mrs. Knight to review Hector’s strategy and reflect on his strategy for solving the problem.

Mrs. Knight said,

Hector is an English language learner, and I’ve found that he becomes very verbal when it comes to math. He will really think outside the box, so I kind of go along with it because I know that’s him just feeling confident about using his language. Because of the language barrier, sometimes he will look for other strategies because he can’t quite put words with what he wants to say.

I then confirmed,

So to clarify, he actually decomposed the 15 into $10 + 4 + 1$. He knew that $10 \times 6 = 60$ from Jackson’s strategy. Then, he used repeated addition to find the product
of 4 x 6 by calculating 6 + 6 = 12 and 6 + 6 = 12. He then used partial sums to find 12 + 12. He decomposed 12 into 10 + 2 and 10 + 2. Then, he added 10 + 10 = 20 and 2 + 2 = 4, and 20 + 4 = 24. He knew that 1 x 6 = 6, so he added 20 + (6 + 4). He added 6 + 4 = 10, and 20 + 10 = 30. He then added 30 + 60 and got an answer of 90.

Mrs. Knight responded,

Yes, that’s correct. I find that he needs to break down computation problems in a way that makes sense to him, and then he’ll build from there. He needs that to be successful, so he’s using a lot of different strategies often.

Throughout the study, Mrs. Knight displayed great patience with her students and encouraged them to explain their strategies in a way that made sense to them. She consistently asked questions such as, “why or why not,” “what is this telling you,” “what do you notice,” and “what’s happening” to seek understanding from her students. Posing questions was a major avenue for getting students to speak and expand on their reasoning for solving problems.

*Figure 26* displays participant themes for teacher questioning.
Figure 26. Participant themes for teacher questioning.

Cross-Case Analysis

In order to make generalizations about the study, it was necessary to find the commonalities and variations among the cases. During the second phase of data analysis, I conducted a cross-case analysis to connect the themes that emerged across the cases (Merriam, 2001). Themes from each case were analyzed and compared across the cases to identify commonalities in the data. The cross-case analysis uncovered the following themes: verbalization of reasoning; increased language and communication skills; kind and respectful learning environment; facilitator who selects purposeful number strings; and student-to-teacher discourse (see Figure 27). Table 9 provides a summary of the cross-case results. The following sections present these findings.
Figure 27. Participant cross-case themes.
Table 9

Summary of Cross-Case Results

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Verbalization of Reasoning</th>
<th>Increased Language and Communication Skills</th>
<th>Kind, Respectful Learning Environment</th>
<th>Facilitator Who Selects Purposeful Number Strings</th>
<th>Student-to-Teacher Discourse</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mrs. Miller</td>
<td>“In number talks, we’re just talking and reasoning about problems. “I feel like number talks takes the pressure off the class because I’m not expecting them to produce anything or turn something in.”</td>
<td>“Their speaking, communication, reasoning, and vocabulary all improved because of number talks.”</td>
<td>“We’re a family, and we have class meetings. We talk about making mistakes, and that mistakes are proof that you are trying, and there’s nothing to be embarrassed about. Overall, they are respectful of each other.”</td>
<td>“Compared to previous years, this group of students has grasped concepts more quickly because of the preview [of number strings] they’re getting through number talks. The number talks are giving them the preknowledge needed.”</td>
<td>“For number talks, we try and keep it quick, so they are supposed to stay quiet and not have any side talk.”</td>
</tr>
<tr>
<td>Mrs. Addison</td>
<td>“As far as the verbalizing and reasoning about problems, I do think that’s coming from the number talks. Once they get the confidence with saying it aloud, they feel more comfortable with writing it down on paper.”</td>
<td>“With the turn and talk, they have a chance to reason through their answer and receive peer feedback before sharing it with the entire class.”</td>
<td>“We’re all here to learn, and number talks is a time for us to share our thinking and learn from each other.”</td>
<td>“At times, I use my assessment data to determine the types of problems that were difficult for students, and then I incorporate similar problems in my number talks so we can review and discuss them.”</td>
<td>“No, I have not tried [the turn and talk], although I am interested in it.”</td>
</tr>
<tr>
<td>Mrs. Knight</td>
<td>“I’m trying to make sure that we’re consistently bridging the language of verbal reasoning to written reasoning.”</td>
<td>“I really see the results of number talks in the language and the vocabulary they use in their explanations. Because of our conversations, they have internalized the meanings of vocabulary words. They are making sense of problems.”</td>
<td>“[Number talks is] a dedicated time where everyone is really paying attention, and it’s a respectful and peaceful time where they are thinking and applying.”</td>
<td>“Sometimes my number strings are based on the content that we are discussing in class because I want to understand how they are reasoning about current concepts.”</td>
<td>“That is one of my weaknesses. I’ve observed the practice of turn and talks through a colleague, but I have not learned to appreciate the usefulness of it, so I definitely underutilize it in practice.”</td>
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</table>
Verbalization of Reasoning

An analysis of the data across the three cases revealed that all participants perceived number talks as a critical component of developing mathematical reasoning in students. Throughout the study, all participants emphasized the importance of students verbalizing their reasoning before producing written responses. They recognized number talks as a means of achieving this objective.

During the interviews, Mrs. Miller remarked on several occasions that number talks allowed her students to reason about problems without the added pressure of producing a written response. She stated, “In number talks, we’re just talking and reasoning about problems.” Mrs. Addison also shared in this perception, remarking that number talks provided opportunities for her students to verbalize their thinking and develop self-confidence prior to constructing a written response. Mrs. Knight, too, stressed the importance of verbalizing reasoning before generating a written response. She declared, “I’m trying to make sure that we’re consistently bridging the language of verbal reasoning to written reasoning.”

Increased Language and Communication Skills

Language and communication emerged as another theme in the cross-case data analysis. All three cases identified number talks as a pathway to increased language and communication skills in students. Specifically, Mrs. Miller and Mrs. Knight recognized the impact of number talks on students who were learning the English language.

During the initial interview, Mrs. Miller stated that a few years ago, she had a group of students who qualified for ESOL services. After implementing number talks as
part of math instruction, she noticed a change in the students’ language skills. Mrs. Miller said, “Their speaking, communication, reasoning, and vocabulary all improved because of number talks.”

Mrs. Knight also highlighted the effects of number talks on students who were English language learners. Mrs. Knight stated, “[My students] are able to fall back on the language that they use in number talks to make sense of problems and talk their way through solving problem.” She continued, “I have several students in this class who are English language learners, and when we’re talking about math outside of number talks, they are using language from number talks. So, I know it benefits them.” Additionally, Mrs. Knight noted the internalization of math vocabulary in her students as the result of number talks.

Mrs. Addison noted the influence of number talks on students’ language and communication during the peer turn-and-talks. She remarked that the student-to-student communication allowed learners to receive peer feedback before sharing their answer with the entire class. This resulted in building self-confidence in their understanding of math concepts.

Kind, Respectful Learning Environment

An analysis of the cross-case data revealed that learning environment was another common theme among participants. All three cases perceived a respectful learning environment as a critical part of implementing effective number talks. Each participant acknowledged the importance of community in creating a safe atmosphere for the sharing of mathematical ideas.
At the start of the study, Mrs. Miller commented that one of her major teaching philosophies was maintaining self-esteem in her students. Mrs. Miller emphasized to her students that mistakes were a part of learning, and as a class, they were supporting each other in their journey together. Mrs. Miller noted that mistakes were not an embarrassment, but proof of the learning process.

Mrs. Addison also emphasized the importance of community during the study. She stated, “We’re all here to learn, and number talks is a time for us to share our thinking and learn from each other.” Mrs. Addison remarked on several occasions that kindness was an important characteristic for students to possess. During one of the observations for this study, a student questioned if others would laugh at her answer if she responded. The student’s fear transpired from an earlier incident in the session where her peers ridiculed her strategy to a problem. Mrs. Addison expressed that she disapproved of this behavior and did not “want anyone to feel intimidated” by their answers. Multiple times throughout the study, Mrs. Addison commented that respect and kindness were essential pieces of the learning environment.

Mrs. Knight described the learning environment during number talks as a “respectful” and “peaceful” time for students. She stated that, during number talks, “we honor each other’s thought process.” Mrs. Knight conveyed to her students that number talks was not a time for competition, but a time to think about the strategies shared by other members of the group.
Facilitator Who Selects Purposeful Number Strings

Prior to the first interview, I asked each participant to describe her role during number talks. All three cases shared the same response: facilitator. In pursuing a broader understanding of the role of facilitator among the cases, I asked each participant to describe the traits of a facilitator during number talks.

Mrs. Miller: Facilitator. Mrs. Miller defined her role of facilitator through multiple responsibilities. As a facilitator, Mrs. Miller established a need to identify number strings that aligned with the skills, ideas, or concepts that she wanted her students to understand. Throughout the study, Mrs. Miller consistently stated that she utilized number strings as a preview to upcoming content in mathematics class.

Another role that Mrs. Miller perceived as the job of the facilitator was that of questioner. Throughout the study, Mrs. Miller continuously asked her students to explain their thinking after each response by asking “why”. If students struggled to explain their reasoning, Mrs. Miller modeled the concept with visuals and manipulatives to support their thinking.

Mrs. Miller also exhibited the importance of recording in her facilitation of number talks. She recorded all student solutions, correct or incorrect, on the board. She followed this with discussions of the solutions to support students in their learning.

Mrs. Addison: Facilitator. Mrs. Addison defined her role through multiple perspectives. As the facilitator, Mrs. Addison selected number strings based on two methods. Number strings were designated by the current content in mathematics class or constructed as the result of formative assessment data.
Another role of the facilitator as defined by Mrs. Addison was that of recorder. Mrs. Addison noted that it was her responsibility to write the students’ solutions on the board and record their strategies. Furthermore, Mrs. Addison discovered through this study that the recording of strategies should be neat and organized to assist students in following the conversation.

Mrs. Addison also remarked that, as the facilitator, she was responsible for wait time allotment needed for students to solve each problem. The amount of time was dependent on the students’ hand signals to indicate their readiness to discuss the problem. Furthermore, Mrs. Addison stated that it was her role to call on students to share their responses and to determine if other students agreed or disagreed with the reasoning.

Mrs. Knight: Facilitator. Mrs. Knight, too, defined the role of facilitator through various lenses. As the facilitator, Mrs. Knight, much like Mrs. Addison, selected her number stings based on current content in mathematics class or formative assessment data.

Mrs. Knight also identified the role of facilitator as that of a recorder. At one point, I asked Mrs. Knight, “How important do you feel it is to intentionally record the strategies of students?”

Mrs. Knight replied,

I think it’s very important, because sometimes I write things on the board, and my students are like “no,” and I love that! I think it really provides those learning moments for them. I love when they tell me, “No, that’s not what I meant,” and they revert back to explain it more precisely.
Mrs. Knight also mentioned that her role as the facilitator was to monitor the amount of wait time provided for students to solve problems. In one of the interviews, Mrs. Knight expressed, “That is where I struggle—how much wait time is appropriate without taking away the think-time needed to solve the problem.” Furthermore, Mrs. Knight felt that as the facilitator, she was responsible for guiding the conversation, although she expressed concerns that her voice sometimes dominated the discussion. Mrs. Knight stated, “I really want to get to the point where I barely hear my voice, and I’m trying to figure out how to balance that during number talks.”

**Student-to-Teacher Discourse**

During the first interview, I asked participants to describe how they promote mathematical discourse during number talks. An analysis of the data revealed that all three participants primarily relied on student-to-teacher discourse during number talks prior to start of this study. During the study, Mrs. Miller always relied on student-to-teacher discourse, stating that student-to-student discourse was typically reserved for the mini-lesson during the math instructional block. During the first interview, Mrs. Addison remarked that student-to-student discourse was not a common practice during number talks, but she expressed interest in implementing the practice. Mrs. Addison found student-to-student discourse engaging for her students, and as a result, she began to incorporate the routine regularly during her number talks. During the first observation for this study, Mrs. Knight incorporated student-to-student discourse into the number talk, but she did not revisit it in other observations. Student-to-teacher discourse was the most prominent form of discourse throughout the course of this study.
Summary

This chapter presented the findings of the role of number talks in constructing number sense, number relationships, and mental computation in third-grade students according to teacher perception. The chapter also provided data findings for (a) the implementation practices of teachers during number talks, (b) the questions that teachers ask to elicit student responses in number talks, and (c) the promotion of student discourse during number talks as reported by teachers. The chapter began with a reiteration of the study purpose and research questions, followed by descriptions of the three participating teachers. A within-case analysis was conducted using preset codes that appeared in the research questions. Finally, a cross-case analysis was conducted to identify themes that emerged across the cases. Chapter 5 presents a discussion of the conclusions, implications, and recommendations of the study.
CHAPTER 5

CONCLUSIONS, IMPLICATIONS, AND RECOMMENDATIONS

This chapter elicits a discussion of the conclusions, implications, and recommendations for future research on classroom number talks. The start of the chapter provides an overview of the study, along with the research questions investigated. The remainder of the chapter reviews the findings, conclusions, and limitations of the research and offers recommendations for further investigation on the topic.

Summary of the Study

The focus of mathematics instruction in the United States has generally centered on rote practices with little emphasis on mental computation strategies (Burns, 2012). Computing problems in a mental capacity encourages students to think flexibly and to reason about numbers (Van de Walle, Karp, & Bay-Williams, 2013). A review of the literature revealed that number sense is the core of understanding in mathematics (Boaler, 2015; National Council of Teachers of Mathematics [NCTM], 2000; Washington, 2015). The intent of classroom number talks is to cultivate number sense and reasoning in students by nurturing fluent thinking through mental computation (Parrish, 2014). The following research question guided this study:

1. What is the role of number talks in developing number sense and number relationships in third-grade students pursuant to teacher perception?

The following subquestions supported the guiding question:
a. How do teachers develop classroom community in number talks?

b. How do teachers perceive their role in classroom number talks?

c. How do mental computation strategies support students with mathematical understanding as observed by teachers?

d. How do teachers select purposeful computation problems for classroom number talks?

e. How do number talks promote student discourse in mathematics as reported by teachers?

f. What questions do teachers ask to elicit student responses during number talks?

This case study examined the role of number talks in the development of third-grade students’ understanding of number concepts pursuant to teacher perceptions. Furthermore, this study investigated how teachers implement number talks in practice, using Parrish’s (2011, 2014) five fundamental tenets: (a) a classroom environment that is safe for the sharing of student ideas, (b) a facilitator that questions instead of telling students, (c) mental math strategies that encourage efficient strategies, (d) purposefully selected computation problems that guide students in developing mathematical relationships and patterns, and (e) discourse that is rich in communicating mathematical knowledge. I also considered the types of questions that teachers asked during classroom number talks to elicit student responses.

This research study incorporated an embedded, multiple case-study design. Three third-grade teachers served as the primary units of analysis, each representing a different
case. Embedded within each case were third-grade student participants, who served as the subunits of analysis. The participants for this study taught at three different elementary schools in a large suburban school district in the southeastern United States. Each setting served a diverse population of students.

Over the course of a six-week period, I conducted interviews and observations to determine how classroom number talks support students in their understanding of number sense and number relationships. The data were coded and analyzed for categories and themes that emerged both within each case and across each case. The following sections offer discussions of the findings and conclusions.

Findings

The results of this study represent teacher perceptions of the role of number talks in developing number sense and number relationships in third-grade students. Moreover, the findings denote how teachers implement classroom number talks in practice. Table 10 compares the findings of this study to Parrish’s (2011, 2014) five tenets of number talks. I conducted a within-case analysis to examine teacher perceptions within each case. I then analyzed the data across the cases to determine themes in the data. The following sections present discussions the findings of the study.
Table 10

Comparison of Study Findings to Parrish’s (2011, 2014) Five Tenets of Number Talks

<table>
<thead>
<tr>
<th>Tenet</th>
<th>Characteristics</th>
<th>Study Findings</th>
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</table>
| Classroom environment and community | • Ideas accepted without judgment  
• Respectful, risk-free environment | • Establishing a kind, respectful environment is critical to the success of number talks. |
| Teacher’s role                | • Facilitator  
• Questioner  
• Listener  
• Learner | • Facilitator  
• Selecting purposeful number strings  
• Establishing appropriate wait time  
• Listening and accurately recording student responses  
• Questioning student thinking  
• Modeling misconceptions |
| Role of mental math           | • Develops number relationships that lead to mathematical fluency | • Number talks encourage mental math strategies, but students do not always rely on the learned mental strategies outside of number talks. |
| Purposeful computation problems | • Intentional problems focused on mathematical relationships | • Selection of number strings based on content preview, content review, and formative assessment data |
| Classroom discussions         | • Explain and investigate number concepts  
• Acquire a toolbox of efficient and flexible strategies | • Student-to-teacher discourse is commonplace during number talks, with little emphasis on student-to-student discourse. |

Number Talks and Number Sense

Number sense is a vital constituent for understanding mathematics (Boaler, 2015; McIntosh et al., 1992; NCTM, 2000; Washington, 2015). The findings of this case study showed that pursuant to teacher perception, number talks support students with
understanding concepts of numbers in various ways. First, when teachers used number talks as a preview of content, they provided the preknowledge and understanding necessary to build student confidence, which directly impacted number sense. This was apparent when Mrs. Miller stated,

These students have a little bit of preknowledge going into a new concept because they’ve had a chance to discuss and talk about it . . . . [Number talks] gives them a little bit more of an understanding going into a new concept, almost like a jumping board.

Second, because number talks are a verbal process, students were able to articulate their reasoning and understanding of number concepts without the stress of written tasks. This was obvious when Mrs. Addison said, “As far as the verbalizing and reasoning about problems, I do think that’s coming from the number talks. Once they get the confidence with saying it aloud, they feel more comfortable with writing it down on paper.” Further support is Mrs. Knight’s declaration: “[My students] are able to fall back on the language that they use in number talks to make sense of problems and talk their way through solving problems.”

Third, using precise language in mathematics is an essential piece of number sense understanding. Teachers asserted that number talks reinforced accurate terminology, which is imperative for effectively communicating knowledge. This was evident when Mrs. Knight expressed,

I really see the results of number talks in the language and the vocabulary they use in their explanations. Because of our conversations, they have internalized
the meanings of vocabulary words. They are making sense of problems, and they are writing and giving examples just from the discussions we’ve had during number talks.

Finally, teachers reported that number talks had a positive influence on English language learners, who received opportunities to verbalize their thinking about problems. For example, Mrs. Miller noted the difference in the language skills of her students who qualified for ESOL services. Mrs. Miller affirmed, “Their speaking, communication, reasoning, and vocabulary all improved because of number talks.” Mrs. Knight also supported these findings when she proclaimed, “I have several students in this class who are English language learners, and when we’re talking about math outside of number talks, they are using language from number talks.”

Number Talks and Number Relationships

Purposeful computation problems afford opportunities for students to recognize the relationships between numbers (Parrish, 2014). The results of this study revealed that teachers intentionally selected number strings that elicited the patterns and relationships that exist between problems. For example, by observing the relationship between the numerator and the denominator, students were able to discern a pattern for fractions that are equivalent to one-half. Mrs. Miller stated, “Number talks are all about patterns and looking for number relationships. Identifying patterns and relationships among and between numbers is really important to developing a strong foundation in mathematics.”
In a different example, students observed the relationship between the factors and products in a series of multiplication equations to distinguish that when one factor is doubled and one factor is halved, the product remains the same. Mrs. Knight reflected, doubling and halving is a strategy that I find very useful, but my students are not very familiar with it. I want them to be able to look for patterns and relationships between numbers, so they have the tools needed to solve problems flexibly.

Number talks is a time for [students] to explore and think about those relationships. By examining the patterns and relationships between problems, students were able to generalize and construct their own knowledge about mathematical concepts.

Classroom Community

The classroom culture establishes a premise for the quality of discourse that occurs among members of a learning community (Hufferd-Ackles, Fuson, & Sherin, 2004). The findings of this case study showed that teachers perceived the classroom learning environment as a critical role in the success of number talks. Mrs. Miller stated, We’re a family, and we have class meetings. We talk about making mistakes, and that mistakes are proof that you are trying, and there’s nothing to be embarrassed about. Overall, they are respectful of each other. They don’t laugh or make fun of each other. One of my major philosophies is maintaining self-esteem, so I never want them to feel like they are not intelligent or anything like that.

Participants described the number talks atmosphere as respectful and peaceful, with kindness identified as an attribute necessary for courteous student interactions.
Mrs. Knight also explained how she developed community by establishing rules for number talks sessions:

[Number talks is] a dedicated time where everyone is really paying attention, and it’s a respectful and peaceful time where they are thinking and applying. It’s the one time when they’re not trying to outdo each other. They’re just really thinking about what they’re saying and analyzing what the other person is saying.

The results indicated that students must feel comfortable and validated in the learning environment. When students sensed their ideas were not valued, they were reluctant to respond and share with the group. An example of this occurred in Mrs. Addison’s class when a student questioned if others would laugh at her response to a problem. The basis for this concern transpired from an earlier incident, where she was laughed at by her peers because of an incorrect response she contributed to the conversation. In response to this situation, Mrs. Addison noted the importance of respect and kindness in the learning environment.

The Role of the Teacher

Parrish (2011, 2014) identified four distinct roles for the classroom teacher during number talks: facilitator, questioner, listener, and learner. The results of this study indicated that teachers undoubtedly viewed themselves as a facilitator during classroom number talks. When asked to describe their role as the teacher during number talks, each participant responded with one word: facilitator. In an effort to define the role of a facilitator during classroom number talks, I asked each participant to describe the characteristics of a facilitator. Facilitator duties identified in the study findings included:
(a) selecting purposeful number strings; (b) establishing appropriate wait time; (c) listening and accurately recording student responses; (d) questioning student thinking; and (e) modeling misconceptions.

Mental Computation Strategies

Researchers acknowledge that mental computation is fundamental in developing students’ number sense (NCTM, 2000; National Research Council, 2001; Parrish, 2011, 2014; Reys, 1985; Reys, 1984). In this study, teachers recognized the role of mental computation in number talks, but students did not always rely on mental strategies to calculate problems outside of number talks according to teacher perception. For example, Mrs. Miller said, “A lot of my students still rely on their fingers when calculating. They don’t always apply the mental strategies we’ve discussed. I think [number talks] encourage mental math, but I don’t always see it carrying over to other situations.”

While the data revealed that a few students were more inclined to rely on mental strategies as the result of classroom number talks, teachers perceived the major success to be students’ ability to explain and reason about problems because of the consistent exposure to number talks. Mrs. Miller declared, “I don’t think number talks have impacted [my students’] mental math abilities as much as just their reasoning. I feel like it’s impacted their reasoning more than their mental math.”

Purposeful Computation Problems

Purposeful computation problems, or number strings, are designed to demonstrate the patterns and relationships that exist between numbers (Parrish, 2011, 2014). The
findings of this study revealed that teachers select number strings using three methods. In one method, teachers selected number strings to preview content before introducing concepts in regular mathematics instruction. The number strings provided the prerequisite knowledge necessary for students to be successful with the content. This was obvious when Mrs. Miller acknowledged,

> Compared to previous years, this group of students has grasped concepts more quickly because of the preview they’re getting through number talks. The number talks are giving them the preknowledge needed to later apply to the problems we do in math class.

The second method that teachers used to select number strings was the current content being addressed during mathematics instruction. Teachers perceived number talks as an opportunity to review content or clarify misconceptions. For example, Mrs. Addison said, “Sometimes I try to line them up with the content that we are currently doing in class, or content that we have previously covered but need to revisit.” After revisiting a problem during classroom number talks, Mrs. Addison reflected on the experience and stated, “I was pleased that this time when they looked at [the strategy], they could automatically see the misconception. When they saw the strategy the other day, they didn’t understand, but this time, they recognized [the error] almost immediately.” Mrs. Knight echoed Mrs. Addison’s remarks on using number talks to review content in saying, “Sometimes my number strings are based on the content that we are discussing in class because I want to understand how they are reasoning about current concepts.”
The third method that teachers employed to develop number strings was formative assessment data. Teachers relied on data to identify content that was problematic for students. The construction of number strings supported students by strengthening the concepts recognized through formative assessment. For instance, Mrs. Addison professed, “At times, I use my assessment data to determine the types of problems that were difficult for students, and then I incorporate similar problems in my number talks so we can review and discuss them.” Mrs. Knight concurred, “Sometimes, I see that my students are struggling with something, and I use number talks as an opportunity to review.”

Student Discourse

Meaningful discourse increases students’ knowledge, understanding, and learning in mathematics (Franke et al., 2009). The results of this study indicated that student-to-teacher discourse was commonplace during number talks. Prior to this study, teachers only engaged in student-to-teacher discourse, but through interviews and conversations that were part of this study, student-to-student discourse became routine practice for one teacher.

As part of this case study, I asked teachers to elaborate on their reasons for not incorporating student-to-student discourse during classroom number talks. Mrs. Miller stated, “For number talks, we try and keep it quick, so they are supposed to stay quiet and not have any side talk.” Similarly, Mrs. Knight said, “That is one of my weaknesses. I’ve observed the practice of turn and talks through a colleague, but I have not learned to appreciate the usefulness of it, so I definitely underutilize it in practice.” Mrs. Addison
provided a different viewpoint, stating, “No, I have not tried [the turn and talk], although I am interested in it.” Over the course of the six-week period, student-to-student discourse became a regular practice in Mrs. Addison’s class. During the final debrief, Mrs. Addison reflected,

I never incorporated the turn and talks prior to this study, and I think it’s a good approach for getting everyone engaged in the conversation. I see more of [the students] getting involved and talking instead of being afraid to share their answer. With the turn and talk, they have a chance to reason through their answer and receive peer feedback before sharing it with the entire class.

Teacher Questioning

The types of questions that teachers pose during instruction affect the responses provided by students (Franke et al., 2009). Data showed that teachers commonly asked two types of questions during number talks: reasoning questions and clarifying questions. When teachers posed reasoning questions, they asked students to explain or justify their thinking. Reasoning questions typically included the word “why” or “explain”, and teachers expected students to elaborate on their thinking.

The other type of question posed by teachers were clarifying questions. Clarifying questions usually required students to repeat segments of their strategy for the purposes of proper recording by the teacher. For example, Mrs. Addison asked clarifying questions such as “You did five, 18 times?”; “Did you add them up as you went along?”; and “Equals how much?” frequently during number talks. These types of questions focused on understanding the process instead of the reasoning behind the process.
Cross-Case Analysis

I conducted a cross-case analysis in order to make generalizations about the findings of the study. Five themes emerged in the data: verbalization of reasoning; increased language and communication skills; kind and respectful learning environment; facilitator who selects purposeful number strings; and student-to-teacher discourse. The findings showed that teachers perceived the verbalization of student reasoning to be a significant part of number talks, as it provided opportunities for students to think and reason about problems before producing a written response. For example, Mrs. Miller stated, “In number talks, we’re just talking and reasoning about problems. . . . I feel like number talks takes the pressure off the class because I’m not expecting them to produce anything or turn something in.” Mrs. Addison agreed, “As far as the verbalizing and reasoning about problems, I do think that’s coming from the number talks. Once they get the confidence with saying it aloud, they feel more comfortable with writing it down on paper.” Likewise, Mrs. Knight said,

I’m trying to make sure that we’re consistently bridging the language of verbal reasoning to written reasoning. . . . Several of my students try to use mental math strategies on their assessments, and it’s great that they are relying on those strategies. But I also need to see the reasoning for their answers, and they don’t always include that. I try to explain that I need evidence of their thinking. In number talks, that’s a verbal conversation, but on assignments, I need to see the written language to understand their reasoning.
Teachers also recognized the benefits of classroom number talks on students’ communication skills. Participating in classroom number talks encouraged students to use precise language and vocabulary to discuss problems. This was evident when Mrs. Miller noted a change in the language skills of her students who qualified for ESOL services. Mrs. Miller stated, “Their speaking, communication, reasoning, and vocabulary all improved because of number talks.” Mrs. Knight also remarked on the effects of classroom number talks on the communication of her students. Mrs. Knight stated, I really see the results of number talks in the language and the vocabulary they use in their explanations. Because of our conversations, they have internalized the meanings of vocabulary words. They are making sense of problems, and they are writing and giving examples just from the discussions we’ve had during number talks.

The results of this study indicated the necessity of a safe learning environment during classroom number talks. Teachers acknowledged the importance of developing a community of learners. Mrs. Knight observed, “[Number talks is] the one time when they’re not trying to outdo each other. They’re just really thinking about what they’re saying and analyzing what the other person is saying.” Mrs. Addison further supported this statement in saying, “We’re all here to learn, and number talks is a time for us to share our thinking and learn from each other.” The findings showed that when students did not feel respected by their peers, they were reluctant to participate.

Teachers unanimously viewed their role during classroom number talks as that of a facilitator. The role of facilitator carried varying responsibilities that consisted of
selecting purposeful number strings, maintaining wait time, recording student responses, and questioning student thinking. Mrs. Miller believed that her role as the classroom facilitator was comprised of selecting number strings, questioning student thinking, and modeling mathematical concepts for students. As a facilitator, Mrs. Addison perceived her role to include selecting number strings, establishing wait time, choosing students to share their thinking, and recording student responses. Mrs. Knight remarked that her role as classroom facilitator consisted of selecting number strings, recording student responses, monitoring wait time, and guiding the conversation.

The findings of this study showed that teachers perceived discourse as an important part of number talks. Student-to-teacher discourse was evident in all situations, while student-to-student discourse was limited. Over the six-week study period, Mrs. Miller always depended on student-to-teacher discourse during classroom number talks, while reserving student-to-student discourse for the mini-lesson. Mrs. Addison, too, relied on student-to-teacher discourse prior to the start of this study. As she engaged students in peer conversations, student-to-student discourse became a routine practice as the result of discussions during this study. Mrs. Knight incorporated student-to-student discourse during the first number talks observation for this study, but it was not evident during the other observations.

The guiding research question for this study investigated the role of number talks in developing number sense and number relationships in third-grade students pursuant to teacher perception. Six subquestions supported the guiding question and focused on how teachers implement number talks in practice. Additionally, the types of questions that
teachers ask during number talks were explored. The following is an overview of answers to the research questions:

- When teachers used number talks as a preview of content, they provided the preknowledge and understanding necessary to build student confidence, which directly impacted number sense.
- Because number talks are a verbal process, students were able to articulate their reasoning and understanding of number concepts without the stress of written tasks.
- Number talks reinforced precise mathematical language, which is imperative for effectively communicating knowledge and developing number sense understanding.
- Number talks had a positive influence on English language learners, who received opportunities to verbalize their thinking about problems.
- Intentionally selected number strings aided in eliciting the patterns and relationships that exist between problems.
- The learning environment should be a safe, respectful space for students to share their thinking.
- The roles of the facilitator consisted of: (a) selecting purposeful number strings; (b) establishing appropriate wait time; (c) listening and accurately recording student responses; (d) questioning student thinking; and (e) modeling misconceptions.
• Number talks encouraged mental computation skills, but students did not always rely on mental strategies to calculate problems outside of number talks.

• Purposeful computation problems were selected in three ways: (a) as a prerequisite to future content; (b) to support current classroom content; and (c) through formative assessment data.

• Student-to-teacher discourse was commonplace during number talks.

• Teachers commonly asked two types of questions during number talks: reasoning questions and clarifying questions.

Conclusions

This section presents the conclusions regarding the role of number talks in developing number sense and number relationships in third-grade students pursuant to teacher perceptions. Also discussed are conclusions drawn from the data regarding the role of classroom community, the teacher, mental computation strategies, purposeful computation problems, student discourse, and teacher questioning during classroom number talks. Figure 28 presents a summary of cross-study findings from previous qualitative research studies compared to the findings of this study.
Figure 28. Comparison of the findings of this study to previous qualitative research of number talks.
Role of Number Talks and Number Sense

While other studies have demonstrated an increase in students’ number sense as the result of classroom number talks (Celski, 2009; Okamoto, 2015; Washington, 2015; Can & Durmaz, 2016), this study contributed to the research by identifying how number talks support students with number sense. The findings of this research study determined that pursuant to teacher perceptions, number talks support students’ understanding of number concepts by: (a) establishing the prerequisite skills needed to build student confidence; (b) verbalizing thought-processes and reasoning; and (c) accentuating precise language and communication.

Role of Number Talks and Number Relationships

Previous studies (Celski, 2009; Clark, 2015; Johnson & Partlo, 2014; Okamoto, 2015; Turner, 2017; Washington, 2015) have not focused on the connection between number talks and number relationships, but the findings of this study suggest that well-crafted number strings prompt students to explore the structures and patterns between numbers and to make generalizations about mathematics. Mrs. Miller demonstrated this when she said, “Number talks are all about patterns and looking for number relationships. Identifying patterns and relationships among and between numbers is really important to developing a strong foundation in mathematics.” Mrs. Knight also supported this claim in stating, “I want them to be able to look for patterns and relationships between numbers so they have the tools needed to solve problems flexibly. Number talks is a time for [students] to explore and think about those relationships.”
Role of Classroom Community

Although the significance of the classroom culture in mathematics has been thoroughly researched (Bennett, 2014; Hufferd-Ackles et al., 2004; Meltzoff, 1994; NCTM, 2000; Yackel & Cobb, 1996), no prior studies (Celski, 2009; Clark, 2015; Johnson & Partlo, 2014; Okamoto, 2015; Turner, 2017; Washington, 2015) have been conducted to show the relationship between the classroom community and its influence on classroom number talks. The results of this study indicated the importance of a learning environment where all students feel safe, respected, and valued in sharing their thinking. Mrs. Miller reported that in a classroom community, all learners support one another. One way she accomplished this is by modeling recognition of achievements and promoting student self-esteem. She stated, “One of my major philosophies is maintaining self-esteem, so I never want them to feel like they are not intelligent or anything like that. So I praise, praise, praise, praise, praise a lot.”

Mrs. Addison focused on kindness to build positive a positive class community:

We have class meetings, just in general, about being kind. If I see them not being nice to each other at any point during the day, I’ll stop what we are doing, and we come to the carpet for a class meeting to discuss it. I always tell my students that I want them to be smart, but I also want them to be kind. Kindness to me is more important than being smart. If you’re smart, but you’re mean to people, where is that going to get you in life? So that’s really big for me—being good, moral people as well as being smart. So, we have lots of class meetings to discuss that goal.
Mrs. Knight discouraged competition by emphasizing respect. She stated that during number talks, “We all come together in one place so that we’re focused in one specific area of the room. We honor each other’s thought process, and there is no racing to get to the answer first.”

Role of the Teacher

The findings of this study revealed that teachers perceive their role during classroom number talks as that of a facilitator. As a facilitator, teachers noted several responsibilities, including the importance of modeling mathematical concepts. Okamoto (2015) found that visual representations aid students in understanding mathematical concepts during classroom number talks. Turner (2017) revealed similar findings in that visual representations and multiple strategies support student learning in number talks. The results of this study resonate with these findings as teachers used both concrete and visual representations to model problems for understanding.

Role of Mental Computation Strategies

Data revealed conflicting views on the role of mental computation strategies in students’ understanding of mathematics. While teachers acknowledged that mental computation was a goal of number talks, they expressed that mental computation was not an approach that most students employed outside of number talks. However, teachers did report that number talks assisted students in developing mental computation skills. Johnson and Partlo (2014) found that number talks: (a) had a positive influence on students’ mental computation skills; (b) increased students’ knowledge of mental computation strategies; and (c) improved students’ accuracy when mentally solving
computation problems. The findings of this study suggest that students inconsistently rely on mental computation strategies pursuant to teacher perceptions, but number talks do encourage students to explore mental strategies.

Although teachers reported inconsistencies with students’ reliance on mental computation to solve problems, teachers unanimously agreed that classroom number talks had a positive effect on students’ reasoning skills. Mrs. Miller emphasized that number talks give students an opportunity to think and reason about problems. Likewise, Mrs. Addison noted that number talks allow students to verbalize their thinking and develop self-confidence. Similarly, Mrs. Knight highlighted the need to verbalize reasoning before bridging to a written response.” These findings are consistent with those of Johnson and Partlo (2014), who reported that because of number talks, students were able to articulate and reason about the strategies they used to solve problems. Clark (2015) also conveyed that number talks promote student autonomy and reasoning in mathematics.

Role of Purposeful Computation Problems

The results of this case study found that teachers selected purposeful computation problems in three ways: (a) as a prerequisite to future content; (b) to support current classroom content; and (c) through formative assessment data. Similarly, Okamoto (2015) suggested that teachers examine student work as a formative assessment to support their learning during number talks. The outcome of this study is parallel with Okamoto’s (2015) findings, in that teachers reviewed formative assessment data to develop number strings that correlated with students’ learning needs.
Role of Student Discourse

Data indicated that student-to-teacher discourse was the principal mode of communication during classroom number talks. Although prior studies have not specifically focused on the type of student discourse and interactions during number talks, several studies have reported an increase in student participation (Celski, 2009; Clark, 2015). For example, Clark (2015) found that student participation in number talks increased when randomly selecting students to share their responses. This is similar to Mrs. Knight’s response when she stated,

I do have students who will hide, and sometimes I tell them, “I’m going to wait until everyone shows me that they have an answer.” Then, they realize they are going to have to come up with a response, and I may call on one of them. And I know they really don’t want to participate, but it’s my way of letting them know that if they participate, they’ll get more comfortable giving answers.

While these findings are not directly comparable, the results show a relationship between student participation, discourse, and number talks.

Role of Teacher Questioning

In this study, teachers typically asked two types of questions during classroom number talks: reasoning questions and clarifying questions. While reasoning questions required students to explain their thinking, clarifying questions only necessitated that students confirm their process for solving the problem. Clark (2015) found that open-ended questions promote student autonomy and reasoning. Those findings are consistent with the results of this case study. This study concluded that when teachers asked
students “why” or to “explain,” they were more likely to expand on their thought processes; whereas with clarifying questions, they were less inclined to share their reasoning. Throughout the study, Mrs. Addison asked a number of questions that required students to restate their thinking. For example, during the second number talks observation, Mrs. Addison introduced the problem $18 \times 5$. Prior to this problem, the students had solved $18 \times 4$. As the session continued, the following vignette unfolded:

Kylie explained, “Well, I know from the last problem that $18 \times 4$ is 72, so then I just added 18.”

Mrs. Addison replied, “You just added 18? And what did you get?”

Kylie answered, “90.”

Mrs. Addison confirmed, “90? Okay, one more strategy!

Instead of asking Kylie to explain why she added 18, Mrs. Addison simply clarified the procedures that Kylie implemented to solve the problem. As a result, Kylie was not forthcoming in justifying her reasoning for adding 18 to find the solution.

Implications

Three implications for instructional practice transpired as the result of this case study. These implications are (a) norms for mathematical discourse; (b) recording of student responses; and (c) change in teacher practices. The following sections offer a discussion of each.

Norms for Mathematical Discourse

Mathematical discourse is one of the five fundamental tenets of classroom number talks (Parrish, 2011, 2014). Number talks are a multifaceted classroom practice.
Students are not only expected to share their strategies for solving mental computation problems, but they are also responsible for presenting constructive arguments to justify their thinking and reasoning (Parrish, 2014). Although teacher-to-student discourse holds several benefits for students, student-to-student discourse is of critical importance in the mathematics classroom (Yackel, Cobb, Wood, & Merkel, 1990; Yackel, Cobb, Wood, Wheatley, & Merkel, 1990). While the development of social norms is a tradition in many classroom settings, the norms established for typical classroom routines are not adequate for mathematical discourse (Yackel & Cobb, 1996).

The results of this study indicated that student-to-student discourse was not commonplace during classroom number talks. Because of these findings, it is recommended that teachers develop sociomathematical norms (Yackel & Cobb, 1996) to advance the levels of mathematical discourse during classroom number talks. As discussed in Chapter 2, Yackel and Cobb described sociomathematical norms as “understanding of what counts as an acceptable mathematical explanation” (p. 461). Sociomathematical norms should be created by the teacher and students together (Yackel & Cobb, 1996) to define specific criteria for acceptable discourse during classroom number talks. For example, a sociomathematical norm might be developed to encourage the inclusion of student-to-student interactions and discourse. Additional details on the development of sociomathematical norms were conveyed in Chapter 2.

Recording of Student Responses

Another implication for instructional practice is the accurate and organized recording of student solutions and strategies. Accurate recording practices are the sole
responsibility of the classroom teacher. For example, during the first observation, Mrs. Knight, in recording Jackson’s strategy for $6 \times 15$ (see Figure 16), wrote $18 + 6 = 24 + 6 = 30$. This is not representative of accurate recording since $18 + 6$ does not equal 30; the meaning of the equals sign was overlooked. Instead, this should be recorded as $18 + 6 = 24$, and $24 + 6 = 30$. Errors such as this have the potential to cause confusion and misconceptions in young learners. Therefore, accurate recording practices are quintessential for nurturing precision in mathematics.

Organized recording is another responsibility of the classroom teacher. All students must be able to follow the strategies and reasoning of their peers, and the intentional recording of strategies assists students in making these connections. Therefore, each strategy and solution should be neatly recorded in an organized manner. If the recording area is limited and difficult to follow, teachers should seek out additional space, such as chart paper, dry-erase boards, or computer resources. After watching the video from the first observation, Mrs. Addison reflected,

There were just little things that I noticed . . . , such as the spacing on the board when recording strategies. I was writing all over the place, and even I was struggling to follow along. It was too jumbled, so I need to make sure that I’m a little neater so [my students] don’t get confused. I need to write clearer so those who don’t understand are able to follow the conversation.

Additionally, it is important that teachers are able to revisit problems or strategies that proved difficult for students. Consequently, the recordings must be organized, legible, and accessible for future discussion.
Change in Teacher Practices

A third implication for instructional transformation is the adjustment in teacher practices as the result of participation in this study. Because teachers were involved in conversations about their teaching, they were responsive to changes in their instructional craft. Additionally, they were able to review videos of themselves teaching, which provided a powerful form of professional learning. At the end of research period, I asked each teacher to share her reflections on participation in the study.

Mrs. Addison stated,

I’ve enjoyed the feedback, and I’ve learned a lot about number talks that I didn’t know before. I ventured out of my comfort zone, and I’m thankful for these sessions and the different ways you’ve had me look at number talks.

Similarly, Mrs. Knight remarked,

I would say this has been really good for me because it has made me rethink how I approach number talks with my students. I had never really considered how powerful a turn and talk could be, but when I was listening to [my students], it just really opened my eyes. They were really using the language and actually talking about what patterns they noticed instead of waiting for me to lead the conversation. The things that we’ve discussed, and the opportunity to immediately apply them, has helped me a lot.

The implications for these findings suggest that when teachers receive opportunities to discuss and review their practices with a colleague, they are motivated to reflect and implement changes in their practice within a short period of time. For a
similar approach to be effective in schools, administrators or academic coaches might employ peer-to-peer observations, feedback protocols, or, with the teacher’s permission, video recording. These recommendations allow teachers to invest in their own professional learning and set personal goals to improve their teaching practices.

**Recommendations for Future Research**

The findings of this case study suggested that students do not always rely on mental computation skills to solve problems outside of number talks. Additional research could investigate the frequency in which students rely on mental strategies learned through classroom number talks as opposed to strategies that require written procedures.

Another opportunity for future research is investigation of the effects of number talks in a small-group setting. All number talks observations for this study occurred in a whole-group setting, and although student-to-student interactions were minimal, it was difficult to interpret conversations that did occur between students. This would be more attainable in a small-group setting. Furthermore, it would be interesting to see how students who were hesitant to participate in a large-group setting would respond in a small-group setting.

This research study investigated how third-grade teachers implement classroom number talks in practice according to the five essential components identified by Parrish (2011, 2014): (a) a classroom environment that is safe for the sharing of student ideas, (b) a facilitator that questions instead of telling students, (c) mental math strategies that encourage efficient strategies, (d) purposefully selected computation problems that guide students in developing mathematical relationships and patterns, and (e) discourse that is
rich in communicating mathematical knowledge. Future researchers could replicate this study to investigate the practices of teachers at different grade levels. Furthermore, little research has been conducted beyond the elementary level, so future studies could explore the impacts of number talks on middle- and high-school students.

Participants in this study addressed the influence of number talks on English language learners several times. Future research studies could examine the impact of classroom number talks on students in different subgroups. For example, researchers might study the influence of number talks on students who qualify for special education or gifted services or students who qualify for ESOL services.

In summary, recommendations for future research include:

• The frequency in which students rely on mental strategies learned through classroom number talks.
• The effects of number talks in a small-group setting.
• Replication of this study to investigate the practices of teachers at different grade levels.
• The impact of classroom number talks on students in different subgroups.

Final Thoughts

The purpose of this case study was to investigate the role of classroom number talks on students’ understanding of number concepts pursuant to teacher perceptions. Additionally, I examined how teachers implement number talks in practice according to the five tenets identified by Parrish (2011, 2014). I also analyzed the types of questions that teachers pose to elicit student responses during classroom number talks.
The findings of the study suggest that classroom number talks influence students’ number sense understanding by encouraging them to verbally reason their thinking. Number talks also promote accurate and precise communication about mathematics. The learning environment is especially important to the success of number talks as students who feel their responses are unappreciated by other students may be reluctant to participate. The role of the classroom teacher is vital to classroom number talks. As the facilitator, the teacher is responsible for determining purposeful computation problems, listening and accurately recording student strategies, and posing questions that engage students in deeper thinking. Student discourse is also a necessity during classroom number talks, although student-to-teacher discourse dominated the sessions that were observed during this study.

Overall, the findings of this study were consistent with the results of other empirical research on classroom number talks. Although some variances existed among studies on the topic, the majority of researchers reported a correlation between classroom number talks and increased number sense in students. This study supports those findings and recognizes that verbal reasoning and communication during classroom number talks contribute to students’ understanding of number concepts. Number talks are the essence for sense making in mathematics.
REFERENCES


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Thursday, September 27, 2018

Ms. Miranda Westbrook
Mercer University
Tift College of Education - Atlanta
3001 Mercer University Dr
Atlanta, GA 30341

RE: Making Sense of Mathematics Through Number Talks [H1809226]

Dear Ms. Westbrook:

On behalf of Mercer University’s Institutional Review Board for Human Subjects Research, your application submitted on 19-Sep-2018 for the above protocol was reviewed in accordance with Federal Regulations 45 CFR 46.104(a) and 45 CFR 46.105(c) for expedited review and was approved under category(ies) O6, O7 per IRB Item 60364.

Your application was approved for one year of study on 27-Sep-2018. The protocol expires on 26-Sep-2019. If the study continues beyond one year, it must be re-evaluated by the IRB Committee.

Item[s] Approved:
New student application for multiple case study designs using semi-structured interviews to investigate the role of the classroom number tasks in developing number relationships, mental computation, and reasoning in elementary mathematics students.

NOTE: You MUST report to the committee when the protocol is initiated. Report to the Committee immediately any changes in the protocol or consent form and ALL accidents, injuries, and serious or unexpected adverse events that occur to your subjects as a result of this study.

We at the IRB and the Office of Research Compliance are dedicated to providing the best service to our research community. As one of our investigators, we value your feedback and ask that you please take a moment to complete our satisfaction survey and help us improve the quality of our service.

It has been a pleasure working with you and we wish you much success with your project! If you need any further assistance, please feel free to contact our office.

Respectfully,

[Signature]

Ava Chambliss-Richardson, Ph.D., OP, CIMP.
Director of Research Compliance
Member
Institutional Review Board

“Mercer University has adopted and agrees to conduct its clinical research studies in accordance with the International Conference on Harmonization’s (ICH) Guidelines for Good Clinical Practice.”

Mercer University IRB & Office of Research Compliance
Phone: 478-301-4101 | Email: ORC_Mercer@Mercer.EDu | Fax: 478-301-2329
1501 Mercer University Drive, Macon, Georgia 31207-0001
Friday, November 30, 2018

Ms. Miranda Westbrook
Mercer University
Truitt College of Education - Atlanta
1302 Mercer University Dr.
Atlanta, GA 30341

RE: Making Sense of Mathematics Through Number Talks [H1809216]

Dear Ms. Westbrook:

On behalf of Mercer University's Institutional Review Board for Human Subjects Research, your Modifications for Expedited Review submitted on 26-Nov-2018 to the above referenced protocol was reviewed and approved on 30-Nov-2018 in accordance with Federal Regulations 46.111 and 46.112 under category[c][1] 07 for expedited review.

Changes Approved:
A modification application for an amendment to the research study: (1) Adding a signature line for the principal of the school on the parent consent form, the participant consent form, and the student assent form. (2) The time frame of the study in the parent consent form, the participant consent form and the student assent form has been adjusted to say “over the next three-to-eight weeks… to allow for flexibility in scheduling observations and interviews. (3) Parent consent form has been translated into Spanish. This version will be distributed to Spanish-speaking parents in the class rooms where observations will take place.

NOTE: The approval date of this modification does not change the annual renewal date of your protocol which expires on 26-Sep-2019

We at the IRB and the Office of Research Compliance are dedicated to providing the best service to our research community. As one of our investigators, we value your feedback and ask that you please take a moment to complete our Satisfaction Survey and help us to improve the quality of our service.

It has been a pleasure working with you and we wish you much success with your project! If you need any further assistance, please feel free to contact our office.

Respectfully,

[Signature]

Ave Chemolisi-Richardson, Ph.D., CIP, CRM
Director of Research Compliance
Member
Institutional Review Board

"Mercer University has adopted and agrees to conduct its clinical research studies in accordance with the International Conference on Harmonizations (ICH) Guidelines for Good Clinical Practice."

Mercer University IRB & Office of Research Compliance
Phone: 478-301-4101 Email: ORC_Mercer@Mercer.edu Fax: 478-301-2229
1171 Mercer University Drive, Macon, Georgia 31207-0001
APPENDIX B

PARTICIPANT INFORMED CONSENT
Making Sense of Mathematics Through Number Talks

Informed Consent

You are being asked to participate in a research study. Before you give your consent to volunteer, it is important that you read the following information and ask as many questions as necessary to be sure you understand what you will be asked to do.

Investigators
Miranda Westbrook, Ph.D. Candidate, Mercer University, Curriculum and Instruction
3001 Mercer University Drive, Atlanta, GA 30341, 678-454-XXXX

Faculty Advisor:
Dr. William Lacefield, Ed.D., Mercer University, Mathematics Education
3001 Mercer University Drive, Atlanta, GA 30341, 678-547-6335

Purpose of the Research
This research study is designed to explore the role of classroom number talks in developing number sense, number relationships, and reasoning in elementary mathematics students.

The data from this research will be used to determine how number talks support students' understanding of mathematics. The results of the study will provide information for my doctoral dissertation regarding the role of number talks in developing number sense, number relationships, and reasoning in third-grade mathematics students. Information gathered during the course of the project will become part of the data analysis and may contribute to published research reports and presentations.

Procedures
If you volunteer to participate in this study, you will be asked to participate in four audio-recorded interviews and three video-recorded observations over the course of three-to-eight weeks. Specifically, you will be asked to participate in an introductory interview and three debriefing interviews following each classroom observation. You will also be asked to participate in three video-recorded sessions of you facilitating classroom number talks. Additionally, you will be asked to share any artifacts, such as lesson plans and anchor charts, that show how you selected the number strings to use with your students. Anchor charts will be reviewed to determine strategies that have been discussed in previous number talks sessions.

Your participation will take approximately one-to-two hours per week over a period of three-to-eight weeks.

Potential Risks or Discomforts
There are no foreseeable risks associated with this study. This research study is not intended to provoke any physical, psychological, emotional, social, or economic discomforts. You may choose whether or not to

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participate in this study. If you volunteer to participate in this study, you may temporarily or permanently withdraw at any time without consequence.

**Potential Benefits of the Research**
As a participant in this study, you will have the opportunity to review video recordings and transcripts of yourself facilitating classroom number talks. This gives you a chance to observe your own teaching and refine your craft as an educator.

Your participation in this study will be beneficial for educational purposes, as it will give insight about the role of classroom number talks in developing number sense, number relationships, and reasoning in third-grade mathematics students.

**Confidentiality and Data Storage**
The researcher of this study will hold all information obtained during the course of the study in strictest confidence. Your name will be removed from the audio and video recordings. Participants and locations will be assigned a coded number or pseudonym to maintain anonymity in transcripts, field notes, artifacts, data analysis, and reporting of results.

Audio and video recordings will be transcribed and analyzed to study the role of number talks in your classroom. The recordings will be used to determine how number talks support students’ understanding of number sense, number relationships, and mental computation strategies in mathematics. Only you, the researcher, and the researcher’s advisor will have access to the audio and video recordings and transcriptions.

Your name will not be associated with individual responses and will be identified only by an assigned coded number or pseudonym. At no time will your name be associated with the results of the research or shared with others. Any identifying information provided by you will never be used as part of the research or associated with the results of the study.

Your responses will be stored in a locked location and will only be used for research purposes by Mercer University. A number or pseudonym will identify the information that I collect from the audio and video recordings and transcriptions of you facilitating classroom number talks. Any documents connecting participant numbers and names will also be kept in separate locked cabinets. All research documentation will be securely stored at Mercer University for at least three years after completion of the study.

**Participation and Withdrawal**
Your participation in this research study is voluntary. As a participant, you may refuse to participate at any time. To withdraw from the study please contact the principal investigator of this study, Miranda Westbrook, by emailing miranda.g.westbrook@live.mercer.edu. Subjects cannot withdraw from the study after data collection has taken place, since all data are anonymous and information will no longer be identifiable as belonging to the participant.

**Questions about the Research**
If you have any questions about the research, please speak with Miranda Westbrook, 678-454-XXXX, miranda.g.westbrook@live.mercer.edu. If you have questions later, you may contact Miranda Westbrook, 678-
Audio or Video Taping
By participating in this study, you agree for the researcher to audio record each interview session and video record each classroom observation. Your name will be removed from the audio and video recordings. A coded number or pseudonym will be used in all transcriptions.

This project has been reviewed and approved by Mercer University’s IRB. If you believe there is any infringement upon your rights as a research subject, you may contact the IRB Chair, at (478) 301-4101.

You have been given the opportunity to ask questions and these have been answered to your satisfaction. Your signature below indicates your voluntary agreement to participate in this research study.

Research Participant Name (Print)   Name of Person Obtaining Consent (Print)

Research Participant Signature   Person Obtaining Consent Signature

Date   Date

Name of School Principal (Print)

School Principal’s Signature

Date
APPENDIX C

PARENT INFORMED CONSENT FORM (ENGLISH AND SPANISH)
Making Sense of Mathematics Through Number Talks

Parent or Guardian Informed Consent Form

Your child is being asked to participate in a research study entitled, Making Sense of Mathematics Through Number Talks. The study is being conducted by Miranda Westbrook, 678-454-XXXX, miranda_g_westbrook@live.mercer.edu under the advisement of Dr. William Lacefield, (678) 547-6335, lacefield_wo@mercer.edu. The results will be used to further my understanding of the role of classroom number talks in developing number sense, number relationships, and mental computation strategies in third-grade students. Your child’s participation is voluntary. A decision to participate in the research will not affect his/her relationship with (_________) Elementary School, his/her relationship with other teachers, or his/her academic standing.

I. Purpose of the Study

This research study is designed to explore the role of classroom number talks in developing number sense, number relationships, and reasoning in elementary mathematics students.

The data from this research will be used to determine how number talks support students’ understanding of mathematics. The results of the study will provide information for my doctoral dissertation regarding the role of number talks in developing number sense, number relationships, and reasoning in third-grade mathematics students. Information gathered during the course of the project will become part of the data analysis and may contribute to published research reports and presentations.

II. Procedures

If you allow your child to volunteer for this study, your child will be asked to participate in number talks, an instructional practice that your child’s teacher regularly implements in her classroom. Your child’s participation will take approximately 15 minutes for each classroom observation over the course of three-to-eight weeks.

Your child will be asked to assent to participate in this research (Assent means that your child will be asked to voluntarily participate in this research). Your child will tell the teacher they want to participate by answering YES or NO after the teacher verbally reads to your child what the research is about and what he or she will be asked to do. As a participant in this study, your child will be video recorded and observed in a whole-group setting as he or she contributes to classroom number talks approximately three times over the next three-to-eight weeks. The video recordings will be transcribed and analyzed to determine how number talks support students’ number sense, number relationships, and reasoning in mathematics.
Parent/Guardians who allow students to participate must:
Give permission for the researcher to analyze video recordings and transcripts of your child’s participation in classroom number talks by reading and signing the consent form.

III. Potential Benefits to Students and/or Society

Some potential benefits for students include: (a) increasing understanding of mathematical concepts; (b) participating in meaningful discussions about mathematics; (c) investigating efficient and flexible strategies for mentally solving computation problems; and (d) reasoning about mathematical problems and solutions.

Your child’s participation in this study will be beneficial for educational purposes, as it will give insight about the role of classroom number talks in developing number sense, number relationships, and reasoning in third-grade mathematics students.

IV. Potential Risks and Discomforts

There are no foreseeable risks associated with this study. This research study is not intended to provoke any physical, psychological, emotional, social, or economic discomforts. Your child may choose whether or not to participate in this study. Students who volunteer to be in this study may withdraw at any time without consequence.

V. Withdrawal of Participation

Your child’s participation is voluntary. Your child will not be penalized or lose any benefits that he/she are otherwise entitled to if you decide that your child will not participate in this research project.

If your child decides to participate in this project, he/she may discontinue participation at any time without penalty or loss of benefits. You have the right to inspect any instrument or materials related to the proposal. Your request will be honored within a reasonable period after the request is received.

VI. Payment for Participation

Students will not be paid for their participation. There is no financial obligation for participants.

VII. Confidentiality and Data Storage

The researcher of this study will hold all information obtained during the course of the study in strictest confidence. Your child’s name will be removed from all video recordings. Participants and locations will be assigned a coded number or pseudonym to maintain anonymity in transcripts, data analysis, and reporting of results.

Video recordings will be transcribed and analyzed to study the role of number talks in your child’s classroom. The recordings will be used to determine how number talks support students’ understanding of number sense, number relationships, and mental computation strategies in mathematics. Only the researcher, the researcher’s advisor, and the classroom teacher will have access to the video recordings and transcriptions.
Your child’s name will not be associated with his or her individual responses and will be identified only by an assigned coded number or pseudonym. At no time will your child’s name be associated with the results of the research or shared with parents or others. Any identifying information provided by your child will never be used as part of the research or associated with the results of the study.

Your child’s responses will be stored in a locked location and will only be used for research purposes by Mercer University. A number or pseudonym will identify the information that I collect from the video recordings and transcriptions of your child participating in classroom number talks. Any documents connecting participant numbers and names will also be kept in separate locked cabinets. All research documentation will be securely stored at Mercer University for at least three years after completion of the study.

Questions about the Research

If you have any questions about the research, please speak with Miranda Westbrook, 678-454-XXXX, miranda.g.westbrook@live.mercer.edu. If you have questions later, you may contact Miranda Westbrook, 678-454-XXXX, miranda.g.westbrook@live.mercer.edu or Dr. William Lacefield, (678) 547-6335, lacefield_woo@mercer.edu.

You have been given the opportunity to ask questions and these have been answered to your satisfaction. If you agree to allow your child to participate in this research, please complete the information below:

I, __________________________, grant my child, __________________________, permission to participate in this research study.

Name of Parent or Legal Guardian

Name of Child Participating in Study

Parent/Guardian Name (Print)

Name of Person Obtaining Consent (Print)

Parent/Guardian Signature

Person Obtaining Consent Signature

Date

Date

Name of School Principal (Print)

School Principal’s Signature

Date

Mercer IRB

Approval Date: 11/26/2018

Expiration Date: 09/26/2019
Please return this form to your child's teacher as soon as possible. Your child's teacher will deliver this form to the researcher, Miranda Westbrook.

In order to conduct this research, this project has been reviewed and approved by Mercer University's Institutional Review Board (IRB). If you believe there is any infringement upon your child's rights as a research subject, please contact the IRB Chair at (478) 301-4101. The IRBs are the governing bodies that are put in place to ensure responsible and safe conduct of research investigations.
Dar Sentido a las Matemáticas a Través de las Conversaciones Numéricas

Formulario de consentimiento informado del padre o tutor

Se le está pidiendo a su hijo que participe en un estudio de investigación titulado, Haciendo Sentido de las Matemáticas a Través de las Conversaciones Numéricas. Este estudio está siendo realizado por Miranda Westbrook, 678-454-XXXX, miranda.gwestbrook@live.mercer.edu bajo el asesoramiento del Dr. William Lacefield, (678) 347-0335, lacefieldw@mercer.edu. Los resultados se utilizarán para mejorar mi comprensión en el papel que desempeña el desarrollo del sentido de las conversaciones de números en el aula, las relaciones numéricas y las estrategias de cálculo mental en estudiantes de tercer grado. La participación de su hijo es voluntaria. La decisión de participar en este estudio no afectará la relación que él o ella tenga con las escuelas elementarias de (________) así como tampoco su relación con otras maestras, o su posición académica.

I. Propósito del Estudio

Este estudio de investigación está diseñado para desarrollar el sentido de las conversaciones en el aula sobre los números, las relaciones numéricas, y el razonamiento de las matemáticas elementarias en los estudiantes.

Los datos de esta investigación se utilizarán para determinar cómo las conversaciones sobre números apoyan el entendimiento de matemáticas en los estudiantes. Los resultados de este estudio proporcionarán información para mi tesis doctoral sobre el papel que desempeña las conversaciones numéricas en el desarrollo del sentido de los números, las relaciones numéricas, y el razonamiento de las matemáticas en los estudiantes de tercer grado. La información recopilada durante el curso de este estudio se utilizará como parte de un análisis de datos y podría contribuir a reportes de informes de investigación publicados y a presentaciones.

II. Procedimientos

Si usted permite que su niño sea voluntario para este estudio, su hijo/hija se le pedirá que participe en conversaciones acerca de números, en prácticas educacionales que la maestra de su hijo/a utiliza regularmente en el aula. La participación de su hijo tomará aproximadamente 15 minutos por cada observación en el aula durante el transcurso de tres a ocho semanas.

Se le pedirá a su hijo que de consentimiento para participar en este estudio (Consentimiento significa que a su hijo se le pedirá que participe en este estudio voluntariamente). Su hijo/a le dirá a su maestro que él o ella quiere participar respondiendo con un Sí o un No después de que la maestra lea verbalmente a su hijo/a de qué se trata el estudio y de las cosas que se le pedirá que hagan. Como participante de este estudio, su hijo/a se grabará en video y se le observará en un ambiente de todo el grupo mientras él o ella contribuye a las conversaciones de números en el aula aproximadamente tres veces durante las próximas tres a ocho semanas. Las videograbaciones se transcribirán y se analizarán para determinar cómo las conversaciones o prácticas acerca de los números apoyan las relaciones numéricas y el razonamiento matemático de los estudiantes.
Los Padres/Guardianes que dan permiso a los estudiantes que participen deben:
Dar permiso para que los investigadores analicen las grabaciones de video y las transcripciones de la participación de su hijo en las conversaciones sobre los números en el aula por medio de leyendo y firmando este formulario de consentimiento.

III. Beneficios Potenciales para el Estudiante y/o la Sociedad

Algunos de los beneficios potenciales para los estudiantes incluyen: (a) aumentar la comprensión de los conceptos matemáticos; (b) participación en conversaciones significativas sobre las matemáticas; (c) investigar estrategias eficientes y flexibles de cómo resolver problemas de cálculo mentalmente; y (d) razonamiento acerca de problemas matemáticos y soluciones.

La participación de su hijo en este estudio será beneficioso para los fines educativos, ya que brindarán información sobre el papel que desempeña las conversaciones de los números en el aula para el desarrollo del sentido común numérico en los estudiantes de matemáticas de tercer grado.

IV. Riesgos Potenciales e Incomodidades

No hay riesgos previsibles asociados con este estudio. Este estudio de investigación no pretende provocar ningún tipo de molestia física, psicológico, social o económico. Su hijo puede elegir si desea o no participar o no en este estudio. Los estudiantes que se ofrezcan como voluntarios para participar en este estudio pueden retirarse en cualquier momento sin consecuencias.

V. Retirarse de la Participación

La participación de su hijo es voluntaria. Su hijo no será penalizado o perderá ninguno de los beneficios o derechos que él o ella tienen si usted decide que su hijo/a no participe en este proyecto de investigación.

Si su hijo decide participar en este proyecto, él o ella puede dejar de participar en cualquier momento que lo desee sin perder ningún beneficio o ser penalizados. Usted tiene el derecho de inspeccionar cualquier instrumento o material relacionado en esta propuesta. Su petición será aceptada dentro de un plazo razonable después de que se reciba dicha petición.

VI. Pago por la Participación

No se pagará a los estudiantes por su participación. Los participantes no tienen ninguna responsabilidad financiera.

VII. Confidencialidad y Almacenamiento de Datos

El investigador de este estudio mantendrá toda la información obtenida durante el curso del estudio en la más estricta confidencialidad. El nombre de hijo/a será eliminado de todas las grabaciones de video. Los participantes y las ubicaciones se les asignará un número codificado o un seudónimo para mantener el anonimato en las transcripciones, los análisis de datos y en el informe de los resultados.

Las videograbaciones serán transcritas y analizadas para estudiar el papel que desempeña las conversaciones de números en la clase de su hijo/a. Las grabaciones serán usadas para determinar cómo las conversaciones de...
numeros apoyan la comprensión de los estudiantes sobre el sentido numérico, las relaciones numéricas, y las estrategias de cálculo mental en matemáticas. Solo el investigador, el consejero del investigador, y la maestra(o) del aula tendrán acceso a las videograbaciones y transcripciones.

El nombre de su hijo no se asociará con las respuestas individuales que su hijo pueda dar y las respuestas se identificarán por un número de código o un seudónimo. En ningún momento el nombre se su hijo se asociará con los resultados de la investigación o compartido con otros padres u otras personas. Cualquier información de identificación proporcionada por su hijo nunca será usada como parte de la investigación o asociada con los resultados del estudio.

Las respuestas de su hijo se guardarán en un lugar seguro y solo la utilizará la Universidad de Mercer para fines de investigación. Un número o un seudónimo se asignará para identificar la información que se recopila durante las grabaciones de video y para las transcripciones de su hijo participando en las conversaciones sobre los números en el aula. Cualquier documento conectando el número de participantes y sus nombres también serán guardados en otros gabinetes bajo llave. Todos los documentos de esta investigación se guardarán en forma segura en la Universidad de Mercer por lo menos tres años después de la finalización del estudio.

Preguntas acerca de esta Investigación
Si usted tiene alguna pregunta sobre esta investigación, por favor de comunicarse con Miranda Westbrook, 678-454-XXXX, miranda.g.westbrook@live.mercer.edu. Si en el futuro usted tiene preguntas, puede comunicarse con Miranda Westbrook, 678-454-XXXX, miranda.g.westbrook@live.mercer.edu o con el Doctor William Lacefield, (678) 547-6335, lacefield_wo@mercer.edu.

Se le ha dado la oportunidad de hacer preguntas y éstas han sido contestadas a su satisfacción. Si usted esta de acuerdo en permitir que su hijo/a participe en este estudio, por favor complete la siguiente información.

Yo, ________________, conozco a mi hijo/a ________________.
 Nombre del Padre/Tutor Legal  Nombre del Niño/a Participante en el Estudio

permiso para participar en este estudio de investigación.

Nombre del Padre/Tutor Legal (Imprenta)  Nombre de la Persona que Obtiene el Consentimiento (Imprenta)

Firma del Padre/Tutor Legal  Firma de la Persona que Obtiene el Consentimiento

Fecha  Fecha

Mercer IRB
Approval Date  11/28/2018
Protocol
Expiration Date  09/26/2019

Rev. January 2017  Page 3
Nombre del Director de la Escuela (Imprenta)

Firma del Director de la Escuela

Fecha

Por favor devuélva este formulario al maestro de su niño tan pronto como sea posible. La maestra de su niño entregará este formulario a la investigadora Miranda Westbrook.

Para llevar a cabo esta investigación, este proyecto ha sido revisado y aprobado por la junta de Mercer University’s Institutional Review Board (IRB). Si usted cree que hay alguna violación de los derechos de su hijo como sujeto a este estudio, por favor de comunicarse con el Presidente del IRB al (478) 301-4101. Los rectores del IRB son organismos encargados de asegurarse de una conducta segura y responsable de esta investigación.
APPENDIX D

STUDENT INFORMED ASSENT
Making Sense of Mathematics Through Number Talks

Informed Assent/Verbal Script
For Children Under 12 Years Old (3rd, 4th, 5th, 6th Graders)

Hello, my name is Miranda Westbrook, and I am a researcher at Mercer University who is trying to learn how number talks support you in learning mathematics.

The purpose of this study is to determine the role of classroom number talks in developing number sense, number relationships, and reasoning in elementary mathematics students.

You are being asked to participate in this study because you are a student in this classroom.

I will be the person in charge of this study, and it will take place in your math class over the next three-to-eight weeks.

What will happen is that I will video record and observe number talks in your classroom three times over the next three-to-eight weeks. In order to keep everything you say or write private, your name will not be used on the items we collect from you. Your name will be removed from the video recordings. Your name will be replaced with made up identification numbers or made up names on transcriptions, tables, and published findings.

Your parent(s) have said that it is okay for you to be in this research study. You do not have to be in this study if you do not want to be. You can change your mind at any time by telling your Mom, Dad, My Assistant, or Me.

_____ NO, I do not want to be in this study.  ____ YES, I want to be in this study.

______________________________  __________________________
Signature of Participant                 Date

______________________________  __________________________
Signature of Person Obtaining Assent    Date

______________________________  __________________________
Signature of School Principal          Date

Mercer University Office of Research Compliance
1501 Mercer University Drive, Macon, Georgia, 31207
Phone: 478-301-4101  Email: ORC_Research@Mercer.edu  Fax: 478-301-2329
APPENDIX E

INTERVIEW QUESTIONS
Mrs. Miller Interview Questions

Introductory Interview

Describe your personal experiences as a math student.

Does that experience impact the way you teach math to your third-grade students?

How many years have you taught elementary mathematics and at what grade levels?

How often do you facilitate number talks in your classroom?

Why do you include number talks in your mathematics instruction?

Describe what a typical number talk looks like in your classroom.

Describe your role as the classroom teacher during number talks.

Describe the traits of a facilitator during number talks.

Describe your process for selecting number strings for classroom number talks.

What steps have you taken to develop a classroom community where all kids feel safe during number talks?

Reflecting on your experiences, how have number talks impacted your students’ understanding of number concepts?

Based on your observations, how has number talks impacted students’ mental computation strategies?

Have you observed students using the strategies introduced during number talks in other class situations?

How do you promote mathematical discourse during number talks?

Do you incorporate any turn and talk during number talks to promote student-to-student discourse?

Is there anything about number talks that I have not asked and you want to share?

Debriefing Interview 1

Share your reflections on the number talks session.

Have you noticed a change in students’ learning of fraction concepts over the past few days as a result of number talks?
Some students appear to be struggling with identifying fractions on a number line. What additional support are you providing during math instruction?

I observed a lot of discourse between you and the students. How do you promote student-to-student discourse during number talks?

Can you provide some additional insight on the conversation regarding “two-tooths,” or two-halves?

What surprised you about the strategies or methods that students produced for equivalent fractions?

Once student consistently reversed the numerator and denominator when naming fractions. For example, he said 12/6 instead of 6/12. What support are you providing this student during math instruction?

Do you have any additional reflections from the session that you would like to share?

Debriefing Interview 2

Share your reflections on the number talks session.

You mentioned that you are using number talks as an opportunity to preview new content. What have you noticed about students’ understanding of fraction concepts as a result of previewing the content through number talks?

What role have number talks played in developing students’ understanding of number relationships?

The focus of the session was equivalent fractions, but the topic quickly turned to fraction inequalities because of the conversation. What big ideas did you want students to understand after the session?

This was the first time that students had experienced fraction inequalities. What role did number talks play in developing understanding of fractional relationships?

During number talks, students are verbally expressing their reasoning and understanding. Have you noticed a correlation to their written assignments?

Do you have any additional reflections from the session that you would like to share?

Debriefing Interview 3

Share your reflections on the number talks session.
During the session, you used both concrete and representational models to represent fraction inequalities. Can you discuss your reasoning for this approach?

Since our observation, you have continued to focus on fraction inequalities. What have you noticed about students’ understanding of fraction inequalities during those sessions?

Some students were struggling to compare fractions with common numerators. What additional support are you providing during math instruction to support students with this skill?

How do you decide on the types of questions you pose during number talks?

What topic do you plan to address with students next week during number talks and why?

Do you have any additional reflections from the session that you would like to share?

Mrs. Addison Interview Questions

Introductory Interview

Describe your personal experiences as a math student.

How has that experience impacted your teaching?

How many years have you taught elementary mathematics and at what grade levels?

How often do you facilitate number talks in your classroom?

Why do you include number talks in your mathematics instruction?

Describe what a typical number talk looks like in your classroom.

As the teacher, what do you think your role is during classroom number talks?

Describe the traits of a facilitator during number talks.

Prior to each number talks session, how do you determine what number strings best meet the needs of your students?

What steps have you taken as a classroom teacher to build a sense of community where all students feel safe to share their responses during number talks?

Reflecting on your experiences, how have number talks impacted your students’ understanding of number concepts?

How have number talks influenced students’ mental computation strategies?

How do you promote mathematical discourse during number talks?
Have you tried incorporating a turn and talk?
Is there anything about number talks that I have not asked and you want to share?

Debriefing Interview 1
Share your reflections on the number talks session.
In our initial interview, you mentioned that you had not previously included a turn and talk, but I noticed you incorporated that during the observation. How have your students responded to this practice?
What are your reflections on including student-to-student discourse during number talks?
Some of the student appeared to struggle with concepts of area and perimeter. What additional support are you providing during math instruction to support students with this skill?
You included a true-false problem as part of your number string. How often do you incorporate these types of problems during number talks?
How do you decide on the types of questions you pose during number talks?
Have you observed students using the strategies introduced during number talks in other class situations?
During number talks, students are verbally expressing their reasoning and understanding. Have you noticed a correlation to their written assignments?
The figure was not drawn to scale in one of the problems. What surprised you about the students’ responses?
Do you have any additional reflections from the session that you would like to share?

Debriefing Interview 2
Share your reflections on the number talks session.
Why did you decide to focus on the distributive property for this session?
After the observation, did you address the students’ behavior that occurred during the number talk?
Do you think that all students feel safe to share during number talks?
What are your perceptions about the strategy that Alice shared?
Some students appear to be relying on the standard algorithm, which is not always conducive to mental math. What have you noticed about students’ use of this strategy?

How do you address the most efficient strategies during number talks?

Have number talks influenced your students’ development of number concepts?

Do you typically have students formulate an estimate before solving problems?

How do you encourage students to compare and contrast the similarities and differences between strategies?

Do you have any additional reflections from the session that you would like to share?

Debriefing Interview 3

Share your reflections on the number talks session.

For this session, you requested that students estimate prior to finding the actual answer. What did you notice or what surprised you in the students’ responses?

Over the last six weeks, you have tried implementing true and false problems and turn and talk during number talks. How has that changed your perception of number talks and its impact on developing student reasoning and number sense?

What were your reflections on revisiting a problem as an error-analysis during number talks?

During the turn and talk, were you able to hear the conversation of the student who originally presented the strategy?

What role did number talks play in clarifying student misconceptions of place value relationships?

Have students struggled with rounding and regrouping only in number talks or also in your regular math class?

Have you modeled regrouping with base-ten blocks during math class?

How do they respond when the manipulatives are in front of them?

You mentioned that you had used number lines for subtraction, but I noticed that none of the students relied on the adding-up strategy to find the difference. What have you observed about students using friendly number relationships to find the solution?

What topic do you plan to address with students next week during number talks and why?

Do you have any additional reflections from the session that you would like to share?
Mrs. Knight Interview Questions

Introductory Interview

Describe your personal experiences as a math student.

In what ways has that experience impacted how you teach math today?

How many years have you taught elementary mathematics and at what grade levels?

How often do you facilitate number talks in your classroom?

Why do you include number talks in your mathematics instruction?

Describe what a typical number talk looks like in your classroom.

What is your role during classroom number talks?

Describe the traits of a facilitator during number talks.

How important do you feel it is to intentionally record the strategies of students?

Describe your process for determining the number strings that best meet the needs of your students.

How would you portray the classroom environment during number talks?

What has been your role in developing that sense of community with your students?

Reflecting on your experiences, how have number talks impacted your students’ understanding of number concepts?

Have you noticed any changes in your students’ understanding of number concepts after regular implementation of number talks with your third-grade students?

How has number talks impacted students’ mental computation strategies in your classroom?

How do you promote student discourse during number talks?

Do you ever have students turn and talk to discuss problems during number talks?

How do you typically respond when a student is struggling to explain the strategy used to solve a problem?

Is there anything about number talks that I have not asked and you want to share?
Debriefing Interview 1

Share your reflections on the number talks session.

Were there some practices that you shifted or adjusted as a result of our initial interview?

Was that the first time that the students have participated in a turn and talk? How did they respond to the practice?

What are your reflections on including student-to-student discourse during number talks?

What were your thoughts on the amount of wait time provided?

For this session, you focused on one problem: 6 x 15. Is there a specific reason you selected this problem?

Can you review and reflect on the recording of Jackson’s strategy?

Can you expand on your reasoning for having students develop a written reflection on Leah’s strategy and Jose’s strategy at the end of the number talk?

Can you review Hector’s strategy and reflect on his strategy for solving the problem?

How many problems do you typically solve during number talks?

If you had continued the number string, what would have been your next problem?

Do you ever model problems for students during number talks?

Do you have any additional reflections from the session that you would like to share?

Debriefing Interview 2

Share your reflections on the number talks session.

Why did you choose to introduce the doubling and halving strategy to students?

What big ideas do you want students to apply after understanding the relationship between doubling and halving?

Students were working on halving even numbers in number talks. You are currently discussing fractions in your regular math class. Have you considered asking students to half an odd number and connect the skill to fractions?

Have you observed students using the strategies introduced during number talks in other class situations?

What topic do you plan to address with students next week during number talks and why?
Do you have any additional reflections from the session that you would like to share?

Debriefing Interview 3

Share your reflections on the session. How did it support students’ development of number sense?

How did the inclusion of visual models support your students with understanding the concept of doubling and halving?

During number talks, students are verbally expressing their reasoning and understanding. Have you noticed a correlation to their written assignments?

How do you address the most efficient strategies during number talks?

What topic do you plan to address with students next week during number talks and why?

Do you have any additional reflections from the session that you would like to share?
APPENDIX F

COMPARISON OF THEMES
### Table F

**Comparison of Themes**

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APPENDIX G

CHAIN OF EVIDENCE
Table G

*Chain of Evidence*

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